OBSERVATIONS AND INTERPRETATION

OF THE POLE TIDE

by

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Abstract

Spectral analysis has been applied to virtually all of the world's long tide records in a search for a signal at the frequency of the Chandler wobble. In Northern Europe the amplitudes are two to ten times greater than equilibrium. Hints of a pole tide are observed at a few other stations, but at most locations the tide does not stand out above the noise continuum. A host of theoretical explanations are rejected because they do not fit the observed large amplitude of the pole tide, and its significant variation along a coast line. In addition, a curious, highly coherent progression of sea level variations to the east across Northern Europe is discovered, with components from DC to 80 days period.

Table of Contents

Table of Contents, cont'd

List of figures

List of tables

Introduction

Conserving angular momentum, a freely rotating body will wobble about its spin axis if its mass distribution deviates from spherical symmetry. If the earth were rigid, this motion would have a period of ten months. The observed period is fourteen months. The "geographical north pole" wobbles to the east around the spin axis of the earth, with an amplitude of about three meters. The oceans, of course, feel the centrifugal force induced by this wobbling. Though small compared to the total mass of the earth, the wobble contribution of the oceans is not negligible. Since they rest on the surface of the earth, their influence on the tensor of inertia is maximized. Furthermore, they are certainly much more free to move in response to applied forces than is the solid earth. Haubrich and Munk (1959) estimate that the equilibrium response of the ocean accounts for about one month of the lengthening of the Chandler period. Motion in the fluid core acts to shorten the period slightly (Jeffreys (1970) p. 284), but the majority of the lengthening is due to the elastic deformation of the mantle.

The most thorough previous study of the pole tide was made by Haubrich and Munk (1959). A large pole tide in the North Sea was reported. The amplitude of the equilibrium tide on an earth with continents was derived and the amount of lengthening of the Chandler wobble period due to the ocean was discussed. The hydrodynamical aspects of the pole tide were not carried very far. It would be very interesting to find such an enhanced pole tide in other locations. The damping of the Chandler wobble of the earth is basically an unsolved problem, as is its excitation, for that matter. It is conceivable that a very non-equilibrium pole tide could account for the damping of the wobble, by dissipation in shallow seas and shelves. This would open up a whole set of interesting problems, involving the rotating earth-ocean system.

Time domain analyses, such as those of Jesson (1964) give only the amplitude, phase and period of a peak. This information must be interpreted with caution, because of the high level of noise at low frequency. For this reason too, work with short records of data, such as in Karklin and Sarukhanyan (1968) may be misleading. Geophysical interpretation of the results of pole tide observations, without the benefit of hydrodynamical discussion and error analysis, has led to some embarrassing statements in the literature.

The present study is composed of two parts. The first reports on an analysis of long records of pole position, sea level and weather. A rather unsuccessful search for enhanced pole tides is made, involving virtually every published tide record more than 25 years in length. However, a detailed analysis of 25 stations in the shallow seas of Northern Europe has given us three results which remain essentially unexplained.

First, pole tide amplitudes up to an order of magnitude greater than the expected equilibrium are encountered. Secondly, a growth of pole tide amplitude along the Dutch coast line is observed, in contrast to the constant amplitude which is expected from the low frequency boundary condition. The context of the pole tide in northern Europe is explored further, in terms of a description of the annual variation in sea level, and the continuum behavior of sea level and weather. The third problem is associated with a highly coherent sea level disturbance, propagating with increasing amplitude to the east across the North Sea and the Baltic, with components from DC to 80 days period. The phase speeds are curiously slow, of order 2 m/sec, across the North Sea and through the Kattegat.

The second part of the study is a hydrodynamical and geophysical discussion of these results. The idea of a resonance at low frequency, due to the excitation of Rossby or topographic standing waves with short length scales, is introduced only to be rejected on rather firm observational and theoretical grounds. Direct tidal forcing of small basins must be rejected, but forcing at a boundary with an oscillating current will provide large amplitudes. Aside from the fundamental problem of the source of such a pole tide current, currents alone cannot explain all the features of the observations. Some other mechanisms are examined, without much success. A gravity wave propagation of the sea level continuum may be evident across the Baltic, but not in the North Sea or the Kattegat, where the observed phase speeds are too slow. Direct forcing by atmospheric pressure has been ruled out, but the potentially more important wind effect has been omitted, due to lack of data. The dissipation of the Chandler wobble itself, in shallow seas, should now be considered a possibility, but precise results still await a proper hydrodynamical model of the pole tide.

Gratitude is expressed to Carl Wunsch, Walter Munk, Myrl Hendershott and Ted Madden for helpful discussion and suggestions. Support was contributed by a NDEA Graduate Fellowship, and by NSF Grant No. GA-32979.

PART I. THE OBSERVATIONS

A. Sources of Data and Methods of Analysis

An extensive collection of world wide tide records has been made by the Permanent Service for Mean Sea Level of the Association d'Oceanographie Physique, sponsored by the IUGG. Values through the year 1964 are to be found in Publication Scientifique number 5, 10, 12, 19, 20, 24, and 26 of that body. World wide records of atmospheric pressure, temperature and precipitation through 1960 are available from a set of volumes, World Weather Records, begun by H. H. Clayton and continued by the U.S. Weather Bureau. Data after 1960 may be obtained from the U.S. Weather Bureau, Monthly Mean Climatic Data for the World. The variation of latitude data is collected by the International Polar Motion Service. Data through 1967 have been published by Walker and Young (1957) and Jeffreys (1968).

The basic strategy in the search for the pole tide is quite simple. The data is key-punched, edited somewhat in the time domain, and fast Fourier transformed. In practice, only long records, of length at least 25 years, may be used to give frequency resolution and statistical reliability to the result. Gaps in the data have been filled by a straight line interpolation. Care must be exercised to account for all changes in datum levels in the tide record, since the low frequency noise contribution of false jumps in the data is quite appreciable.

The large annual component, about 6 to 10 cm in amplitude, produces undesirable side band effects. It may be removed in the time domain by subtracting the mean of each same-named month from the data. That is, the average January sea level over all the years of data is subtracted from each individual January level, and similarly for February, March, etc. Before transforming, the mean is removed, as well as a linear trend, as determined by a best least squares fit.

The observed sea level, ζ, can be considered as a combination of a deterministic signal of amplitude s with an uncorrelated random noise process of amplitude n. Then the powers are additive, so that the observed power is

$$\zeta^2 = s^2 + n^2$$

Correcting for the noise bias and approximating one standard deviation error bars, the estimated of the amplitude of the signal is

$$s = \sqrt{\zeta^2 - n^2} \pm n/\sqrt{2}$$

The phase of the signal is unbiased by the noise, and has a standard deviation of about ± 17 degrees for the signal to noise amplitude ratio of 3 to 1, which is typical for the pole tide in Northern Europe. The error bar analysis is discussed in Wunsch (1967), following the detailed study of Middleton (1960), Chapter 9.

A common method of comparing two time series is the coherency estimate. Given the complex Fourier transforms

$F_1(\omega)$, $F_2(\omega)$ from time series 1 and 2, the complex coherency is

$$\text{coh}_{12} = \frac{\langle F_1 F_2^* \rangle}{\langle |F_1|^2 \rangle^{\frac{1}{2}} \langle |F_2|^2 \rangle^{\frac{1}{2}}}$$

where $\langle\ \rangle$ denotes averaging over a band of frequencies. For example, identical series would have a coherency amplitude of 1, and phase of zero, and uncorrelated series would have a coherency near zero and random phase. We say near zero because random noise biases the estimation procedure. How significant is a particular estimate of coherency? That depends on the number of frequencies that make-up the averaging band. We would like to know just how large a coherency value is necessary, in order to say with 95% confidence, that the coherency is not due to random noise. Turning to the tables of Amos and Koopmans (1963), we find that value to be .44 if we average over 15 frequencies, and .88 if we can only use 3 frequencies. The expected values of the coherency amplitude are .23 and .53 for the wide and narrow bands, respectively. Similarly, the standard deviation of an estimate is .01 and .05, respectively. The error bars for the coherency phase, based on random noise estimates as well, are obtained from Jenkins and Watts (1968), Figure 9.3. The reader is referred to that text for a detailed discussion of coherency estimation.

Thanks to Walter Munk, an edited 84-year-long tide record from Delfzijl, on the Dutch coast, was made available. The data, originally recorded in three-hour increments, was averaged over twelve hours. The resulting 61360 point time series was fast Fourier transformed. The results near the pole tide frequency are virtually identical with those obtained from the monthly mean data. The noise levels and peaks from the original record are about 3 per cent higher than from the monthly record. The noise level is .64 cm at low frequency, and .39 cm in the neighborhood of the fortnightly tide. A large tide is observed at 14.764 days period, amplitude 2.69 ± .28 cm. This is the frequency of the MS_f tide, as well as the difference frequency of the two largest tides, S_2-M_2. Under linear behavior, one would expect a much smaller amplitude, only .08 times as large as M_f, the fortnightly tide. Thus the observed tide at 13.660 days, which is also the period of the difference frequency K_2-M_2, may be due to the non-linear interaction of the large tides K_2-M_2, rather than to a linear lunar fortnightly tide. No peak at M_m, 27.55 days, appears above the noise. The situation in the North Sea is thus in contrast to the islands of the Pacific, where no such highly enhanced MS_f component is observed (Wunsch (1967)).

For the purposes of the pole tide analysis, the use of original records does not appear to be an advantage. Records from Vlissingen 1911-62, Cuxhaven 1922-63, and Honolulu 1905-57 were also analyzed. Of course, the behavior of the long period tides, and of continuum at periods shorter than 2 months are not available from the monthly mean data.

B. The Pole Tide

1. The forcing function

The relationship between the earth's pole position and variations in sea level are to be studied. In this comparison it is desirable to use an actual record of the pole position, expecially in view of the fact that the amplitude of the Chandler wobble varies over decades and has a power spectrum spread over a band of frequencies, rather than just a line. In this section we shall review the derivation of the equilibrium tide from the pole position, and discuss the actual spectra of the astronomical data and the computed equilibrium pole tide.

The centrifugal force induced by the wobble may be described by the potential (Munk and MacDonald (1960))

$$U = -\Omega^2 R_e^2 \sin\theta \cos\theta \, (m_1 \cos\lambda + m_2 \sin\lambda)$$

where Ω is the spin rate of the earth with radius R_e, and where θ is the colatitude and λ the longitude of the position on the earth's surface. The direction cosines between the instantaneous spin axes and the geographical axes are denoted by m_1 and m_2, in the sense that $m_1 = x/R_e$ and $m_2 = -y/R_e$. The conventional pole position coordinates (x, y) are in the directions (Greenwich, 90 deg. W). If the ocean responds to this force in an equilibrium fashion, the sea surface remains an equipotential, at a height

$$\bar{\zeta} = (1 + k - h)\ U/g$$

with respect to the sea bottom. The Love number term $(1 + k - h) \cong (1 + .29 - .59) = .7$ represents a reduction of amplitude due to the yielding of the earth. For example, an equilibrium pole tide of .35 cm, appropriate for the North Sea, would be associated with a rise of the ocean bottom of $hU/g = .30$ cm, while the sea surface rises by the amount $(1 + k)\ U/g = .65$ cm with respect to the center of the earth.

Spectra of the variation of latitude data, m_1 and m_2 for the years 1900-64, are shown in Figure 1. The amplitudes and periods are summarized in Table 1, which includes the values for the equilibrium pole tide, as computed for Delfzijl in the North Sea.

The slightly elliptical annual wobble is fairly well understood as the response to atmospheric forcing. The high pressure zone over Asia in the winter is particularly important. However, at the Chandler frequency, the non-seasonal power of the atmospheric forcing function is not strong enough to excite the wobble. It falls short by two orders of magnitude for a low wobble Q of order 10, and one order of magnitude short for a high Q of order 100. For the details of the analysis, see Munk and Hassan (1961), and also Munk and Groves (1952).

The Chandler wobble is almost circular, with an amplitude of about 10 centiseconds of arc, or a radius

3 meters at the pole. The m_2 component lags behind the m_1 by about 90 degrees, just as expected for a circular motion from west to east. The 65 year length of our time series allows us to concentrate on four harmonics in the vicinity of .84 cycles per year. The most significant feature is that while there are amplitudes of about 10 csec at both the 439.6 and 423.9 day periods, there is a gap in between with an amplitude of only about 2 csec, at the period 431.7 days.

The statistical reality and the geophysical consequences of this gap are somewhat in dispute at the present time. With the use of an extended data series covering the 125 years from 1846.0 to 1971.0, Gasoschkin (1972) suggests that the splitting of the wobble is indeed real, with periods of 428 and 437.5 days. Based on the assumption that the wobble oscillates at a single frequency, previous estimates (Jeffreys (1968)) of the damping time are in the range of one to two decades, or a Q of 30 to 60. This value is much lower than the Q of the earth's free oscillations, which is of order several hundred, where the damping is due to anelasticity in the mantle. Electromagnetic coupling at the core-mantle interface and damping in the ocean have been sought as low Q sinks of wobble energy, without much success. Gaposchkin estimates that a Q greater than 100 is consistent with the two narrow peaks in the spectrum, so that low frequency anelasticity may be the damping agent for the wobble after all. A high Q would reopen the excitation problem as well, since in the past seismic and atmospheric excitation functions were thought to be too weak to keep a low Q wobble excited.

There is very little power in the continuum of the polar motion. In the sea level analysis, we define the noise continuum level by taking the average power over the band .5 to 1.5 cycles per year, omitting the Chandler and annual frequencies. The pole tide in the ocean never has a signal to noise amplitude ratio greater than 4, but the astronomical data enjoys a ratio of about 13, when the noise is estimated in the same fashion.

Components with an amplitude of about .5 csec arc are found at the sum of the Chandler frequencies and the annual frequency, and at twice the annual frequency, but not at twice the Chandler frequencies. These components stand out above their neighbors with an amplitude ratio of 2 or 3 to 1. Any components at the difference frequencies are lost in the rise of noise at low frequency, to a level of 1 or 2 csec arc.

TABLE 1

SPECTRAL SUMMARY OF THE FORCING FUNCTION

<u>Amplitude</u>	.815 cpy 447.9 days	.831 cpy 439.6 days	.846 cpy 431.7 days	.862 cpy 423.9 days	1.000 cpy 365.25 days	
Variation of latitude data						
m_1 (csec arc)	5.22	9.69	2.53	10.09	8.78	$\pm.55$
m_2 (csec arc)	5.22	10.25	2.33	9.63	7.06	$\pm.55$
Calculated equilibrium tide						
Delfzijl (cm)	.18	.35	.09	.36	**	$\pm.04$
<u>Phase lead</u>						
m_1 over m_2 (deg)	96.5	92.6	106.2	91.9	98.1	$\pm \simeq 8°$

** denotes annual removed in the time domain before transforming.

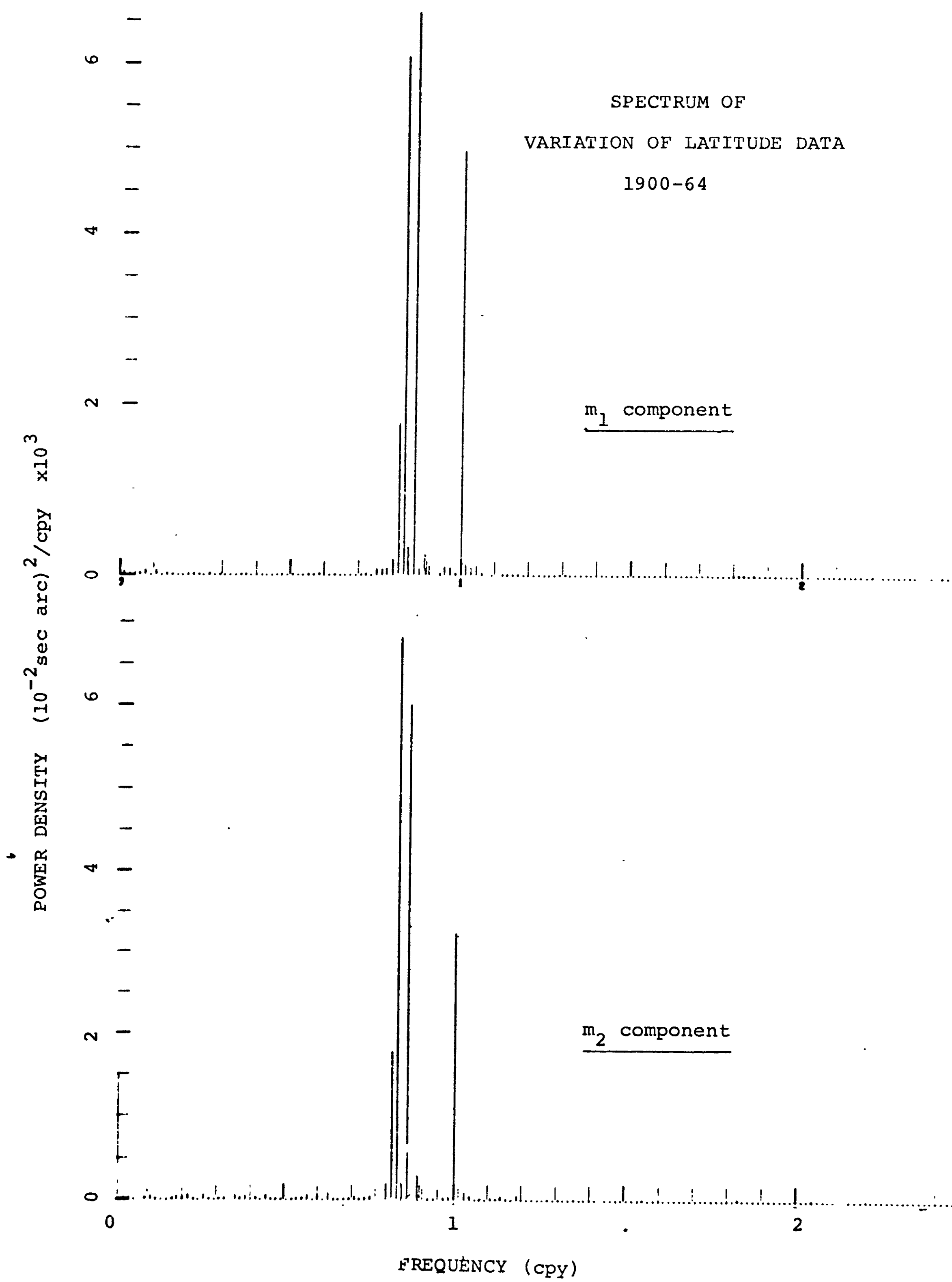

SPECTRUM OF
VARIATION OF LATITUDE DATA
1900-64
m_1 component
m_2 component
POWER DENSITY $(10^{-2}\text{sec arc})^2/\text{cpy}$ $\times 10^3$
0
2
4
6
0
1
2
FREQUENCY (cpy)

Figure 1

2. The Search for the Pole Tide in the World's Oceans

In this study, the behavior of the pole tide in all the world's oceans has been sought. More than 65 mean monthly tide records have been analyzed, each at least 25 years long. The results are basically negative. In general, the amplitudes of the sea level variations at the frequency of the Chandler wobble do not stand out above the noise background. Evidence of an enhanced pole tide is more strong in shallow water than deep, but it is only in the shallow seas of Northern Europe that the pole tide stands out clearly above the noise continuum. Table 2 summarizes the results, as expressed by the average amplitudes of the pole tide and of the noise for each area of study. The signal levels averaged over the Atlantic and over the Pacific are only about 5% higher than the noise, averaged in the same fashion. The amplitudes used have been corrected for the noise bias, so this does not mean that the signal is much smaller than the noise. Rather it means that the signal just happens to be comparable to the noise, and hence the statistical uncertainty of the amplitude of the signal is quite large. Approximate one standard deviation error bars on the pole tide, some for Northern Europe, are typically of order half the amplitude itself.

The results from the 17 stations in the Atlantic Ocean are summarized in Table 3. Whereas large pole tides are observed in the North Sea, on the east coast of Great Britain, no pole tide appears above the noise at Newlyn, facing the

Atlantic. A pole tide seems to be present on the coast of Portugal, perhaps enhanced above the equilibrium. A periodogram from the record at Cascais is plotted in Figure 2. The periodograms of other records are quite similar to this one. The pole tide in the world's oceans lacks the robustness which is evident only in Northern Europe, as is noted by a glance at Figure 5 below. A fairly large pole tide of .77 ± .38 cm appears in the Canary Islands; at the Azores, the amplitude is closer to what is expected from equilibrium response. Buenos Aires, on a wide continental shelf, is below the noise level. Quite a large tide, 1.32 ± .50 cm, is found at St. John in the shallow Bay of Fundy, but not at nearby Halifax, facing the open Atlantic. The pole tide may also be present at Charlottetown, in the shallow Gulf of St. Lawrence. Pole tides of order .5 ± .3 cm are found on the east coast of the U.S., at Portland, Boston and New York. Atlantic City is below the noise level, Baltimore is more like equilibrium, and Charleston is fairly large, .88 ± .44 cm. The pole tide appears to be equilibrium at Pensacola and Galveston, on the Gulf of Mexico.

The results from 6 stations in the Pacific Ocean are summarized in Table 4. On the west coast of America, Ketchikan has a large tide (.98 ± .54 cm), Seattle is about equilibrium, and San Francisco may be slightly enhanced, as is Balboa in the Canal Zone of Panama. Honolulu may be enhanced, with about .6 ± .3 cm as compared to .25 cm for the equilibrium tide. The tide at Sydney is certainly no greater than equilibrium.

The results from 10 stations in a variety of locations throughout the world are summarized in Table 5. In the shallow Mediterranean, there is evidence of an enhanced pole tide at Marseille, but equilibrium prevails at Genoa. A pole tide appears at Aden, at the head of the shallow Gulf of Aden. The shortest record used, 25 years, reveals a large but uncertain pole tide 1.29 ± .72 cm at Churchill in Hudson's Bay. The almost completely enclosed Black Sea, situated near the maximum of the envelope of the pole tide, has an amplitude of .72 ± .41 cm. Like all the other so-called pole tides observed in the world's oceans, this component does not really stand out above the noise. Thus no definitive statement can be made about the ability of the pole tide to excite a closed basin directly. Bombay is below a somewhat small noise level, .44 cm. The records in the shallow Gulf of Thailand are not long enough to substantiate the hint of an enhanced pole tide. The previous results of Haubrich and Munk (1959) are basically consistent with those obtained in the present analysis.

TABLE 2

AVERAGE AMPLITUDE IN WORLD OCEANS

	pole tide (cm)	noise (cm)
North Sea	1.27	.57
Kattegat	1.18	.49
Baltic Sea	3.13	1.16
Atlantic	.56	.53
Pacific	.50	.48

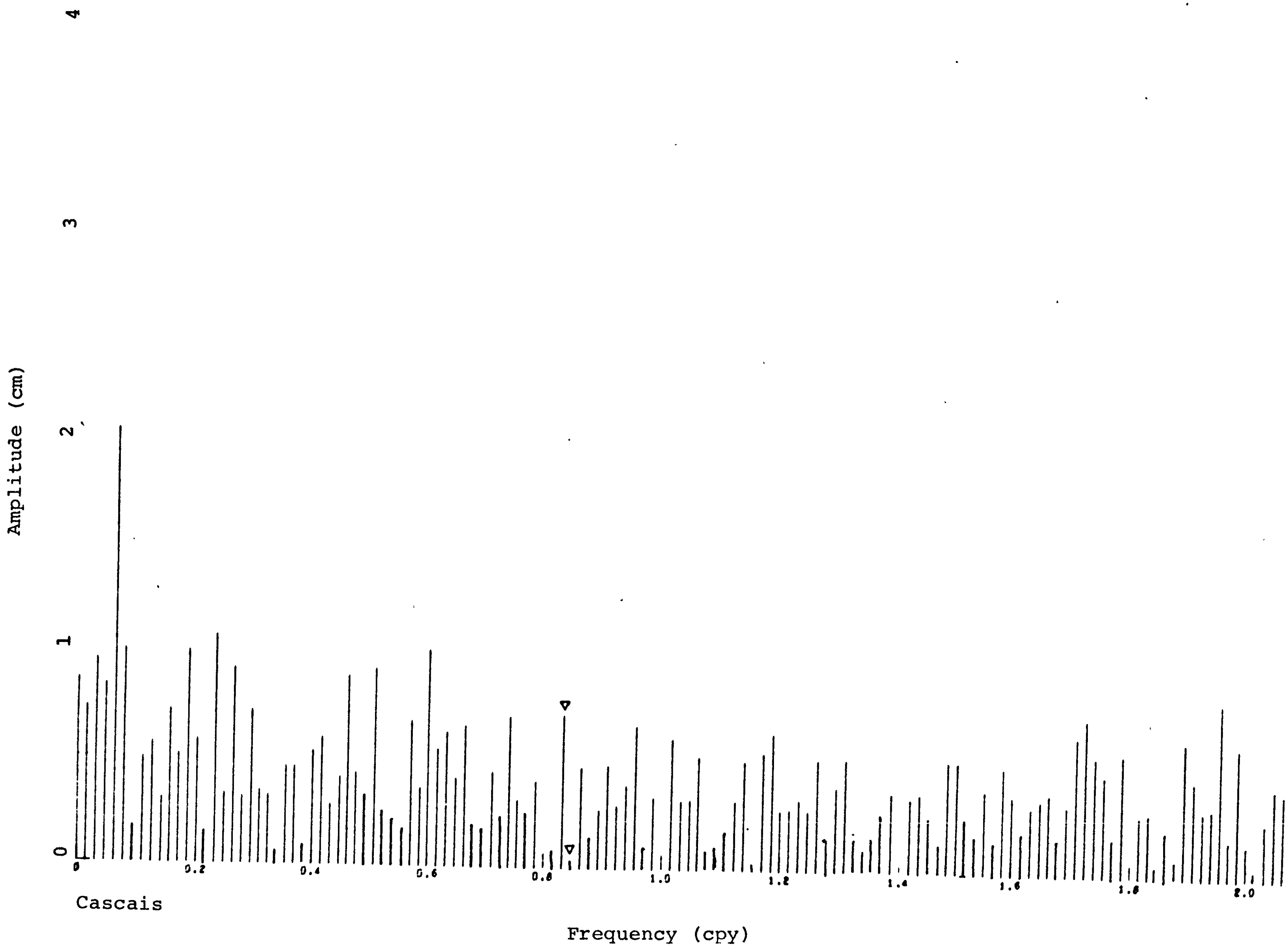


Figure 2 Periodogram of Sea level, Cascais

TABLE 3

POLE TIDE IN THE ATLANTIC OCEAN

		amp. (cm)	per. days	noise amp. (cm)	lat.	lon.
Newlyn	1916-64	bn		.55	50°06'N	5°33'W
Cascais (Portugal)	1900-64	.58 ±.30 .22	439 424	.43	38 41	9 25
Lagos (Portugal)	1909-62	.34 ±.45	438	.64	37 06	8 40
Santa Cruz de Tenerife (Canary Is.)	1927-64	.77 ±.38 .41	434 421	.54	28 29	16 14
Ponta Delgada (Azores)	1930-59	.20 ±.54 .35	438 421	.77	37 44	25 40
Buenos Aires	1905-43	bn		.70	34 36S	58 22
Charlottetown (P. Edward Is.)	1938-64	.64 ±.39	429	.56	46 14N	63 35
Halifax	1920-64	bn		.42	44 40	63 35
St. John	1938-64	1.32 ±.50	429	.70	45 16	66 04
Portland	1924-64	.63 ±.30 .53	440 428	.42	43 40	70 15
Boston	1924-64	.43 ±.31 .66	440 428	.44	42 21	71 03
New York	1900-64	.60 ±.28	424	.39	40 42	74 01
Atlantic City	1912-64	bn		.47	39 21	74 25
Baltimore	1903-64	.27 ±.28 .35	435 427	.40	39 16	76 35
Charleston	1924-64	.88 ±.44	440	.62	32 47	79 55
Pensacola (Gulf of Mexico)	1924-64	.09 ±.35	440	.49	30 24	87 13
Galveston (Gulf of Mexico)	1924-64	.26 ±.55 .48	440 428	.49	29 19	94 48

"bn" denotes "below noise"

TABLE 4

POLE TIDE IN THE PACIFIC OCEAN

		amp. (cm)	per. days	noise amp. (cm)	lat.	lon.
Ketchikan	1924-64	.98 ±.54	428	.76	55°20'N	131°38'W
Seattle	1900-64	.07 ±.33	439	.47	47 36	122 20
		.30	432			
		.40	424			
San Francisco	1900-64	.43 ±.27	439	.38	37 48	122 28
		.61	432			
		.26	424			
Balboa (Panama Canal Zone)	1909-64	.60 ±.29	435	.41	8 58	79 34
		.69	426			
Honolulu		.24 ±.33	439	.47	21 18	157 52
		.56	431			
		.66	421			
Sydney	1900-64	.13 ±.27	439	.38	33 51S	151 14E
		.10	432			
		.29	424			

TABLE 5

POLE TIDE IN "OTHER OCEANS"

Marseille (Mediterranean)	1900-63	1.05 ±.36 .70 .45	441 433 425	.50	43°18'N 5°21'E
Genova (Mediterranean)	1884-1909	.19 ±.54	432	.77	44 24 8 54
	1928-63	.43 ±.60	438	.85	
Aden (Gulf of Aden)	1916-64	.24 ±.37 .51	437 426	.52	12 47 44 59 E
Churchill (Hudson's Bay)	1940-64	1.29 ±.72	435	1.01	58 47 94 12 W
Port Tuapse (Black Sea)	1917-64	.72 ±.41	428	.81	44 06 39 04 E
Bombay (Arabian Sea)	1900-64	bn		.44	18 55 72 50
Bangkok Bar (Gulf of Thailand)	1926-64	.24 ±.62 1.48	432 419	.88	13 27 100 36
	1940-64	.20 ±.61	435	.86	
Phrachuap Kirikhan (Gulf of Thailand)	1940-64	.98 ±.39	435	.55	11 48 99 49
Ko Sichang (Gulf of Thailand)	1940-64	bn		.74	13 09 100 49

3. The Pole Tide in Northern Europe

A large amplitude pole tide in the North Sea has been reported previously by Haubrich and Munk (1959). Let us see what information lies in a detailed analysis of all the available long tide records in northern Europe. In particular we want to know the amplitude and phase relationships between the equilibrium and the observed pole tides, and the amplitude and phase variations of the observed tide across northern Europe.

Let us compare the observed pole tide with the equilibrium, as computed from the astronomical data over the same period. Figure 3 shows a portion of the periodogram of observed and equilibrium sea level at Delfzijl. The sea level shows no sign of the controversial splitting, which characterizes the variation of latitude spectrum. The amplitude of the observed tide is about 1.7 cm, more than 5 times larger than the equilibrium.

We may compute the coherency between equilibrium and observed tides at Delfzijl. Trading off statistical reliability to gain frequency resolution, we average in bands of 3 frequencies. At the pole tide frequency, the coherency amplitude is .68. Strictly speaking, that is below the level of 95% confidence that coherency is not due to random noise. However, the coherencies are much lower, at frequencies away from the pole tide, of order .4, which is about the expected value for uncorrelated signals with 3 degrees of freedom in the estimate.

SEA LEVEL AT DELFZIJL

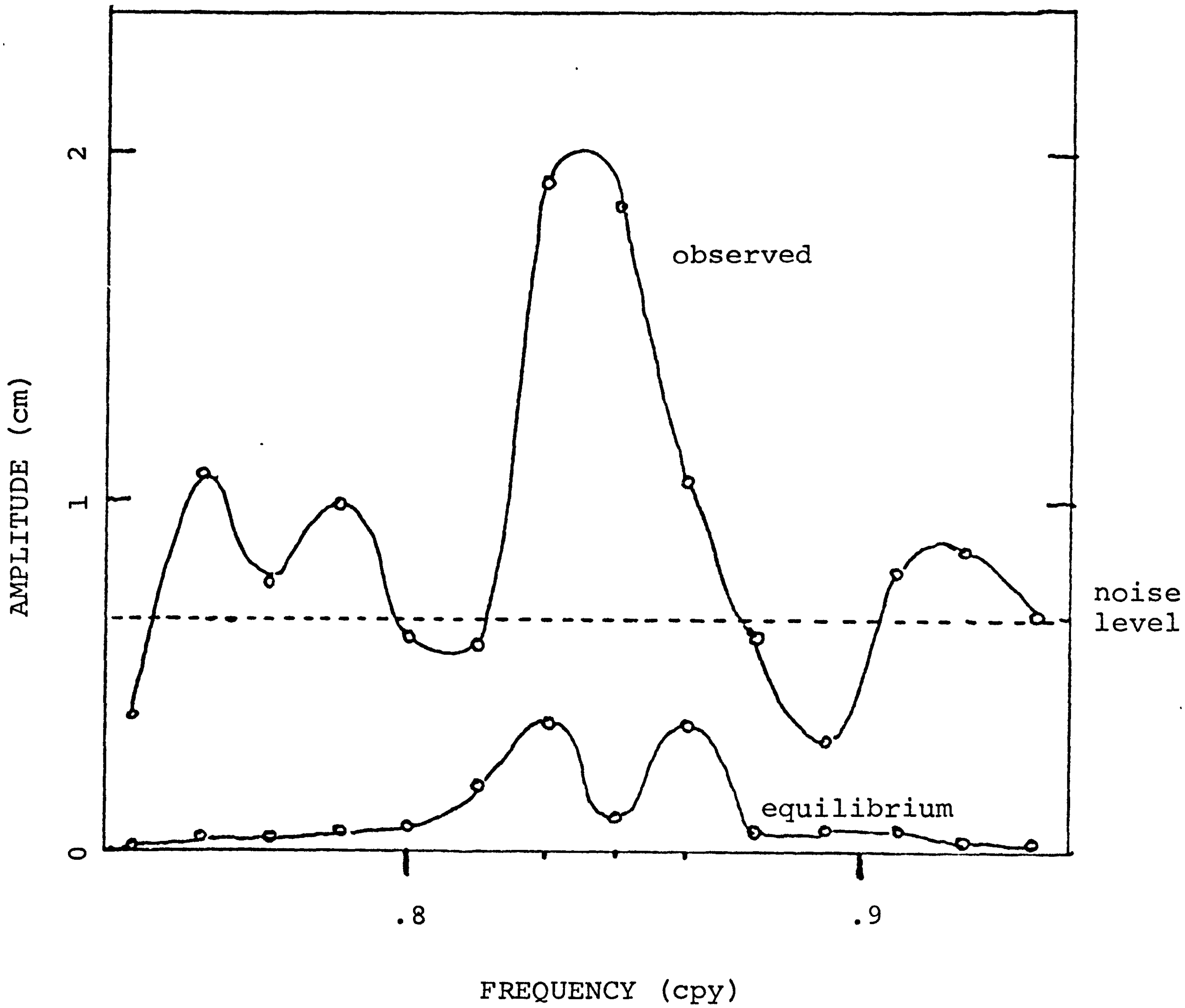


Figure 3   Comparison of observed and equilibrium pol

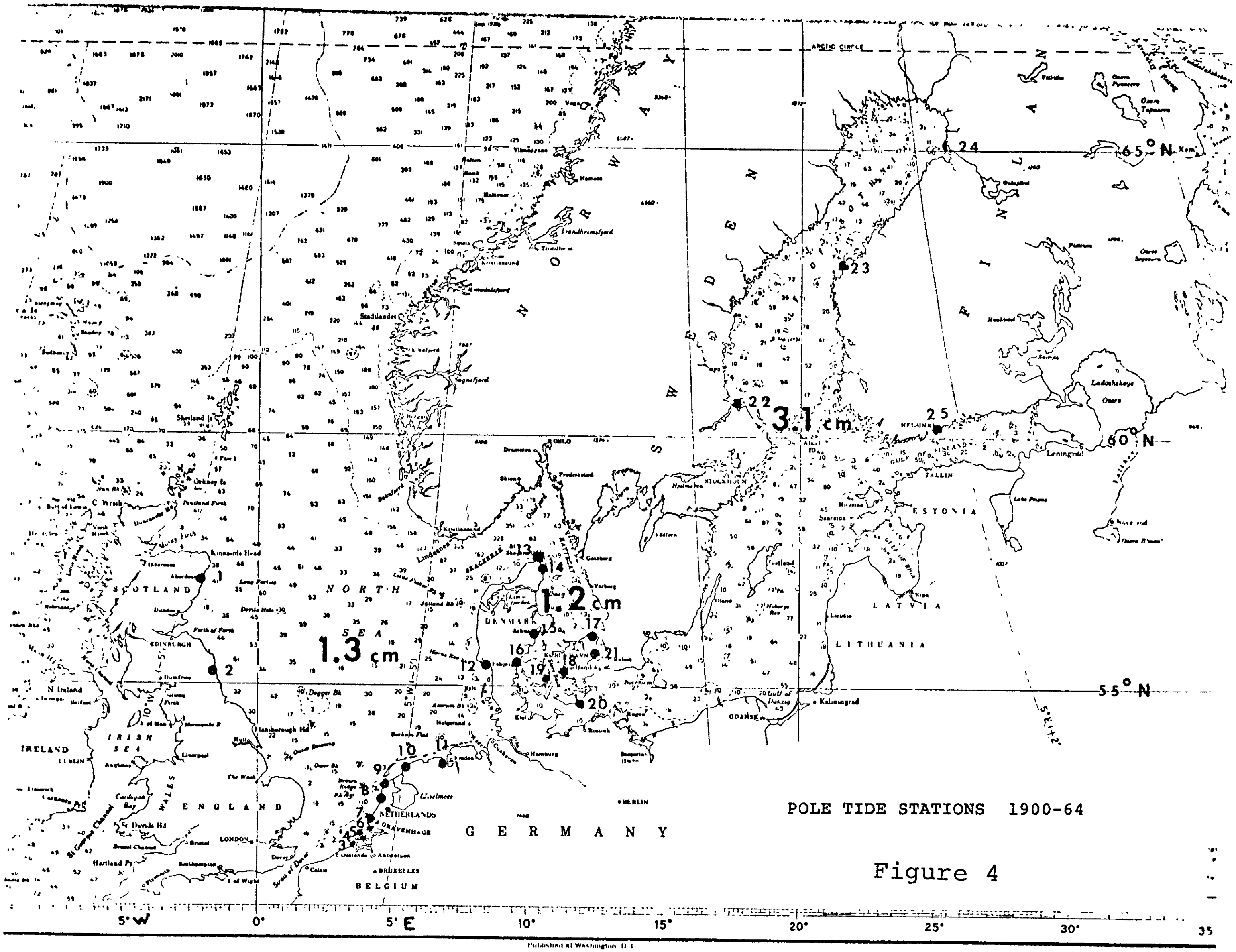
The coherency phase gives us the result that the observed tide lags behind the equilibrium by 52 degrees at Delfzijl. The significance of that delay is questionable. Although the error bars have not been computed explicitly, extrapolation of Jenkins and Watts (1968, Figure 9.3), would suggest that 95% confidence error bars of order ± 40 degrees would be appropriate. The actual phase differences at each frequency do not clear up the situation. The poor signal to noise ratio in the ocean, 2 to 1, gives error bars of order ± 25 degrees. The observed tide lags equilibrium at the 439 and 424 day periods by 80 and 20 degrees. At the equilibrium "gap" at 432 days, the two tides are in phase.

The 126 year long record, Swinemunde 1811-1936, does not have a real peak at the 438, 434,430 or 426 day period. The amplitudes are .2 to .5 cm. However, outside this band, at 442 and 422 days, peaks are found with amplitudes of 1.09 and .83 cm respectively. The noise level is .63 cm, so with this low signal to noise ratio one must be very cautious in ascribing a significant splitting of the pole tide peak. Furthermore, a splitting of both the annual and the Chandler peaks is observed in certain Swedish and Japanese records. The contrast to nearby Danish and Finnish records, with normal peaks, suggests that this effect in Sweden is spurious and due to a data handling problem, as does an unphysical change in sign of the mean trend of sea level, centered about 1937.

There is no evidence of non-linear pole tide behavior in the ocean, since no peak appears above the noise at twice the Chandler frequency, or at the sum or differences of the annual and Chandler frequencies.

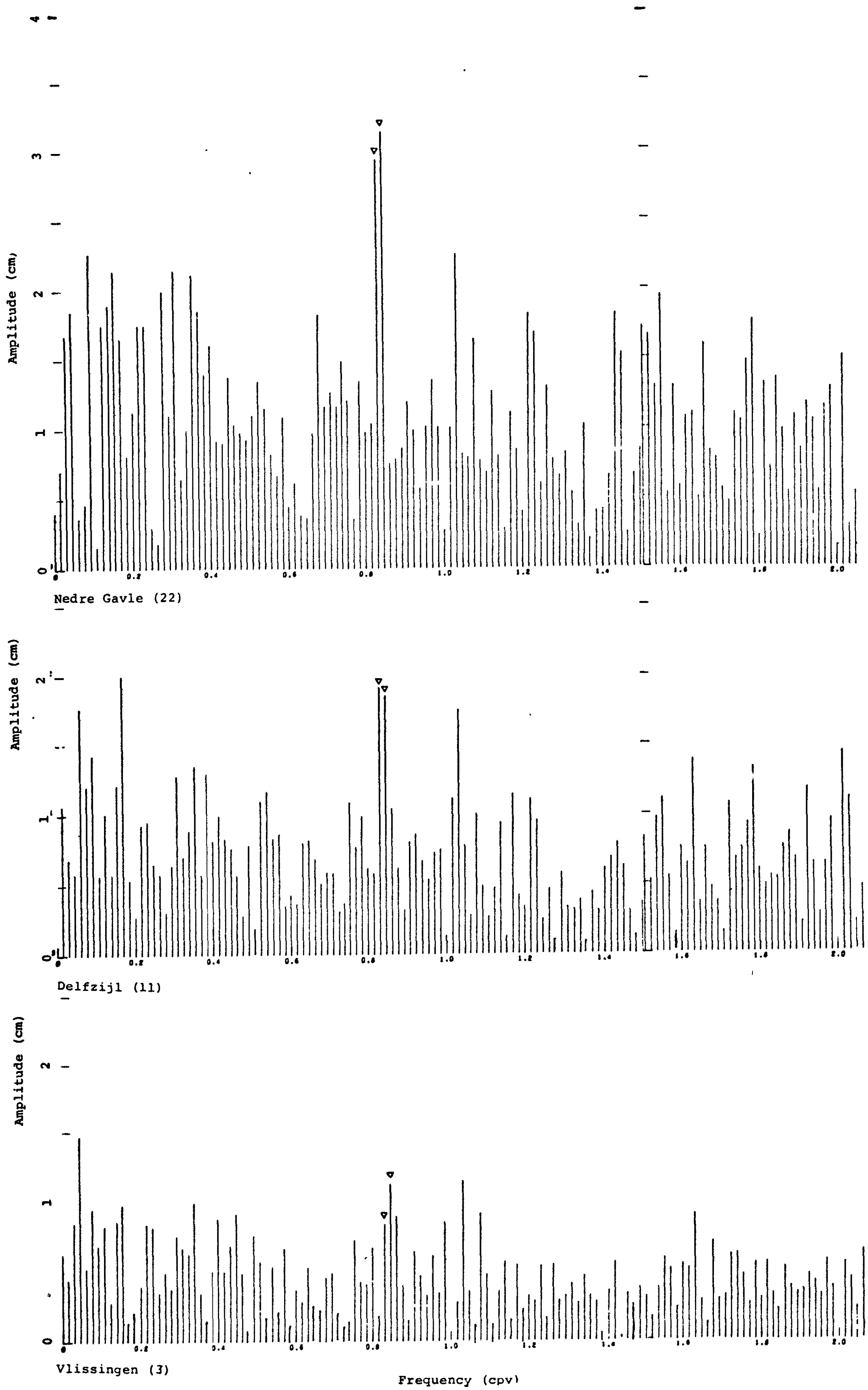
How does the pole tide behave across northern Europe? To answer this question, a collection of twenty-five monthly mean records are used, all covering the years 1900-64. Twelve stations are in the North Sea, nine in the Kattegat area of Denmark, and four in the Baltic Sea, with its adjacent gulfs. A map of the area is shown in Figure 4, which locates each station according to the index numbers of Table 6. Table 6 provides a detailed summary of the pole tide amplitude, phase difference and background noise levels across the region. The average pole tide amplitude for the North sea, the Kattegat area, and the Baltic are also shown on the map. The largest tide is found in the Baltic, where the average amplitude is 3.13 cm, about ten times greater than the equilibrium value. The average in the constricted Danish waters is 1.18 cm. The average value over the North Sea is 1.27 cm, but there is a remarkable growth from .73 to 2.75 cm (at 439 days period) from Vlissingen (3) at the southern end of the Dutch coast to Esbjerg (12) on the western coast of Denmark.

To begin with, let us examine three representative periodograms (Figure 5), over the range 0-2 cycles per year, or DC to 6 months period. A definite growth in the amplitude of the pole tide can be seen, from Vlissingen (3) at the south-



POLE TIDE STATIONS 1900-64

Figure 4



igure 5 Representative periodograms of sea level, northern Europe

TABLE 6

POLE TIDE IN NORTHERN EUROPE 1900-64

		Amplitude (cm)			Phase lead over Helsinki			Noise (cm)	lat.	lon.
	Period (days):	439	432	424	439	432	424			
	North Sea									
1	Aberdeen	1.10	.42	.33 ± .34	52	16	39	.48	57°09N	2°05W
2	North Shields	.55	.57	bn ± .31	62	7	47	.43	55 01	1 24W
3	Vlissingen	.73	1.04	.79 ± .31	44	2	69	.43	51 27	3 26E
4	Zierekzee	.79	.92	.83 ± .33	35	10	73	.47	51 38	3 55
5	Brouwershavn	.88	1.13	1.04 ± .33	33	-5	64	.46	51 44	3 54
6	Hellevoetsluis	.67	1.14	.94 ± .38	28	3	60	.54	51 49	4 08
7	Hoek van Holland	.72	1.09	.49 ± .36	35	21	69	.52	51 59	4 07
8	Ijmuiden	1.14	1.24	.72 ± .40	39	7	63	.57	52 28	4 35
9	Den Helder	1.36	1.46	.45 ± .39	24	2	66	.55	52 58	4 45
10	Harlingen	1.64	1.70	.52 ± .48	16	4	64	.68	53 10	5 25
11	Delfzijl	1.79	1.73	.82 ± .47	22	2	68	.67	53 20	6 56
12	Esbjerg	2.75	2.55	bn ± .63	19	7	41	.88	55 28	8 27

TABLE 6 (cont'd)

POLE TIDE IN NORTHERN EUROPE 1900-64

		Amplitude (cm)			Phase lead over Helsinki			Noise (cm)	lat.	lon.
	Period (days):	439	432	424	439	432	424			
	Kattegat and Zealand									
13	Hirtshals	1.94	1.90	bn ± .49	19	9	42	.69	57°36N	9°57E
14	Frederikshavn	1.41	1.49	.27 ± .34	25	-4	42	.48	57 26	10 34
15	Aarhus	.88	.99	.31 ± .25	16	-15	52	.35	56 09	10 13
16	Fredericia	.44	1.02	.38 ± .25	6	-39	58	.35	55 34	9 46
17	Hornbaek	1.32	1.71	bn ± .45	6	-2	51	.64	56 06	12 28
18	Korsor	.77	1.21	.10 ± .33	-2	-25	54	.47	55 20	11 08
19	Slipshavn	.86	1.09	.28 ± .31	3	-23	51	.43	55 17	10 50
20	Gedser	1.11	1.64	.36 ± .40	-19	-27	42	.56	54 34	11 58
21	Kobenhavn	1.34	1.58	bn ± .43	0	-10	41	.61	55 41	12 36
	Baltic area									
22	Nedre Gavle	2.74	2.97	.73 ± .74	5	3	4	1.04	60 40	17 10
23	Vaasa	2.73	3.31	.85 ± .85	-1	8	-9	1.21	63 06	21 34
24	Oulu	3.26	3.68	.64 ± .85	2	-3	-13	1.21	65 02	25 26
25	Helsinki	2.81	3.52	.31 ± .82	0	0	0	1.16	60 09	24 58

TABLE 7

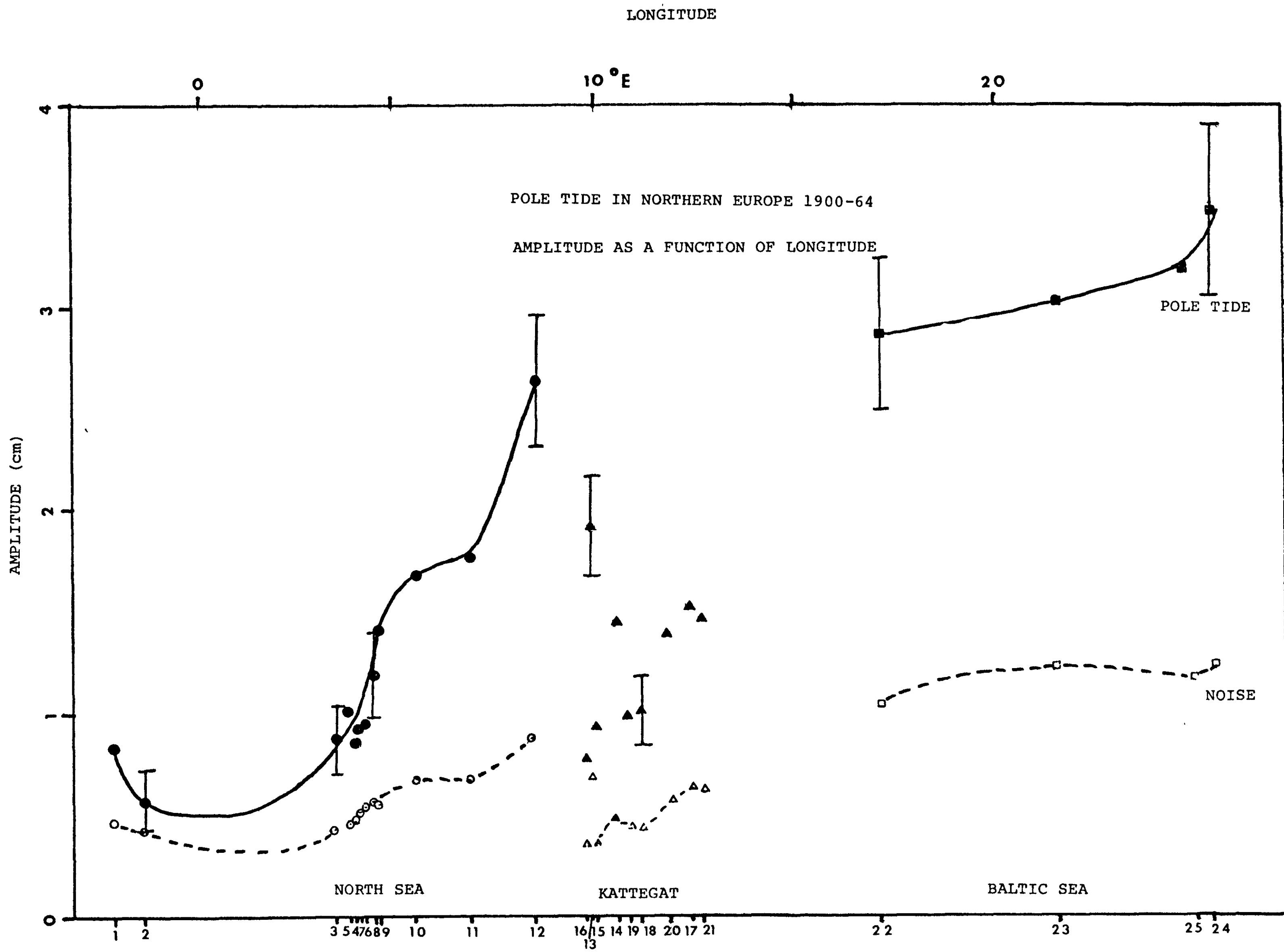
POLE TIDE IN GULF OF BOTHNIA 1924-64

		Amplitude (cm)		Phase lead over Kemi		Noise (cm)	lat.	lon.
	Period (days):	440	428	440	428			
A	Kemi	2.86	3.34 ± 1.14	0	0	1.61	65°44N	24°33E
B	Oulu	3.74	3.73 ± 1.11	-4	-6	1.57	65 02	25 26
C	Raahe	2.92	3.28 ± 1.09	2	-2	1.54	64 42	24 30
D	Pietarsaari	2.91	3.25 ± 1.05	1	-4	1.47	63 42	22 42
E	Vaasa	2.55	3.13 ± 1.08	-5	3	1.53	63 06	21 34
F	Mantyluoto	2.66	3.58 ± .98	5	-1	1.39	61 36	21 29
G	Turku	3.06	3.37 ± 1.01	0	-3	1.43	60 25	22 06
H	Degerby	3.04	3.19 ± .94	-1	-6	1.34	60 02	20 23
I	Nedre Gavle	2.74	3.04 ± .94	2	-1	1.33	60 40	17 10

ern end of the Dutch coast, to Delfzijl at the northern end. The amplitude is even larger at Nedre Gavle in the Gulf of Bothnia, adjoining the Baltic on the north. The background noise grows in the same fashion, as does the annual peak. The annual peak, about 3-5 times larger than the pole tide, does not appear in these periodograms, because it was filtered out in the time domain to avoid side band contamination. The annual term is discussed in detail in a later section.

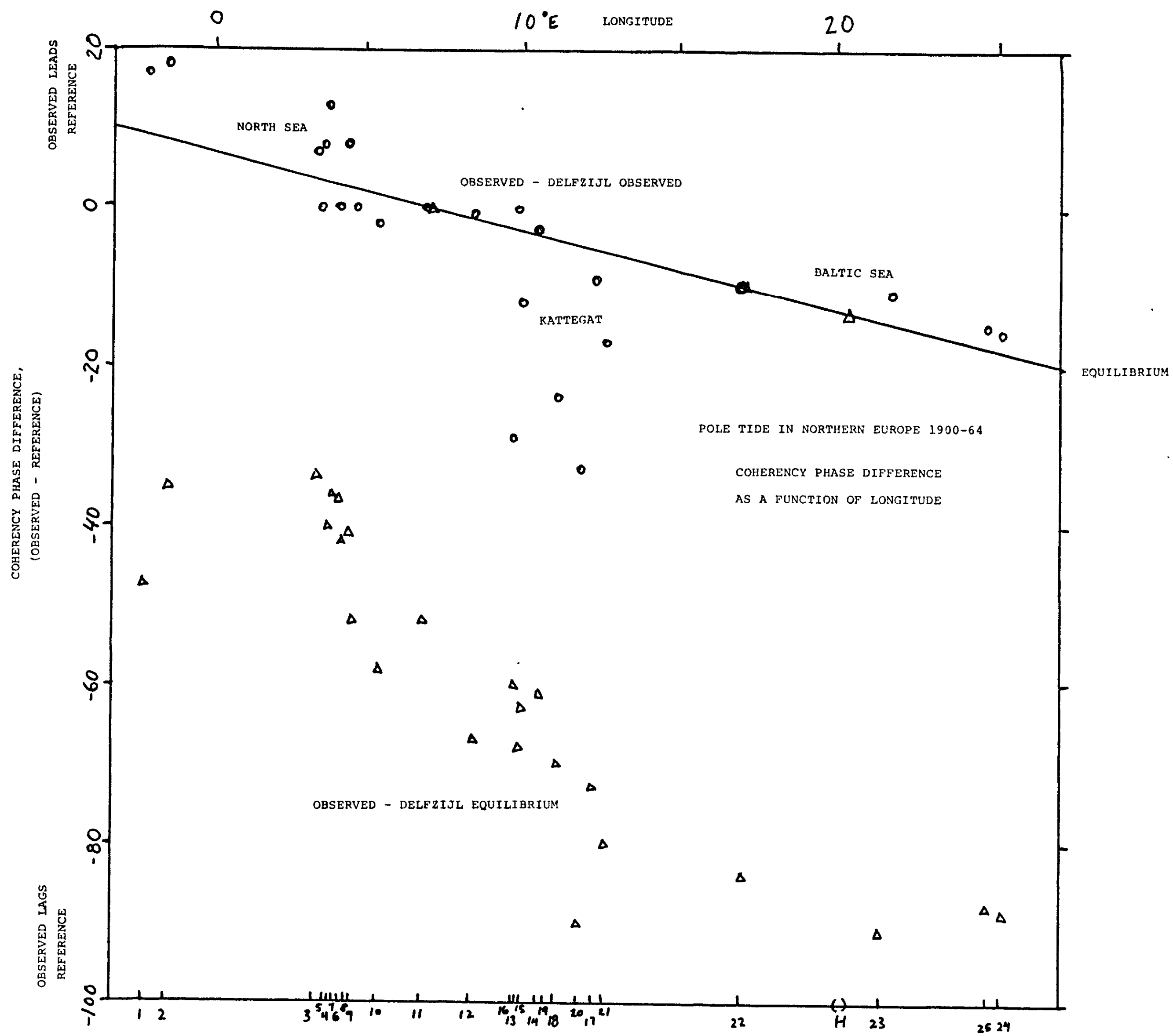
The results of the pole tide observations are most clearly summarized in Figure 6 which plots the amplitude of the average power at 439 and 432 day periods, as a function of the longitude of the station. The striking growth up the Dutch coast, and the large amplitude in the Baltic are the major new results. The amplitude in the constricted waters of the Kattegat and surrounding Zealand are reduced somewhat, but still 2-5 times larger than equilibrium. The background noise behaves in a fashion similar to the pole tide. This suggests that some factors which locally affect the low frequency sea level continuum may also be at work on the pole tide.

Figure 7 displays the phase information of the pole tide in northern Europe as a function of longitude, as obtained by coherency phase estimates based on two different references, Delfzijl equilibrium and Delfzijl observed sea level. The coherency phase difference between equilibrium tides at Degerby (H) and Delfzijl (13 degrees) gives us the variation of the equilibrium tide across northern Europe, (solid line in Figure 7). It is encouraging that this computed phase lag



LONGITUDE
0
10 °E
20
POLE TIDE IN NORTHERN EUROPE 1900-64
AMPLITUDE AS A FUNCTION OF LONGITUDE
AMPLITUDE (cm)
0
1
2
3
4
POLE TIDE
NOISE
NORTH SEA
KATTEGAT
BALTIC SEA
1 2 3 5 4 7 6 8 9 10 11 12 16 15 13 14 19 18 20 17 21 22 23 25 24

Figure 6



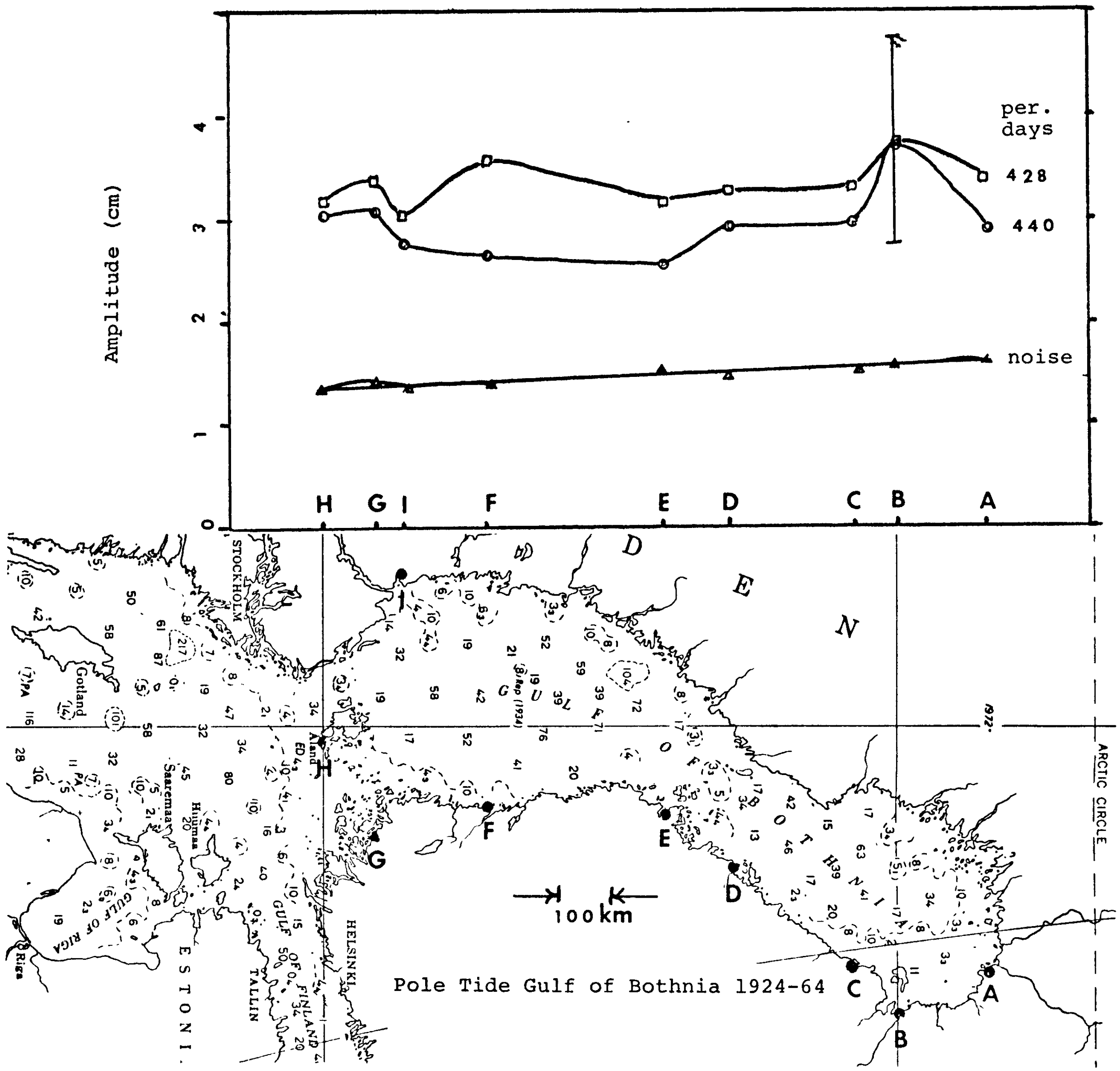
LONGITUDE
0
10°E
20
OBSERVED LEADS REFERENCE
COHERENCY PHASE DIFFERENCE, (OBSERVED - REFERENCE)
OBSERVED LAGS REFERENCE
20
0
-20
-40
-60
-80
-100
NORTH SEA
OBSERVED - DELFZIJL OBSERVED
BALTIC SEA
KATTEGAT
EQUILIBRIUM
POLE TIDE IN NORTHERN EUROPE 1900-64
COHERENCY PHASE DIFFERENCE
AS A FUNCTION OF LONGITUDE
OBSERVED - DELFZIJL EQUILIBRIUM

Figure 7

is equal to the longitude difference of the two stations. That is the expected result for a tide circling the globe from west to east. The coherency phase difference of observed minus equilibrium is plotted (triangles in Figure 7) for each station. Across northern Europe, the observed pole tide continues to be delayed behind the equilibrium by about 50-60 degrees, except for a scattering of larger phase lag in the constricted Kattegat area. Any more detailed interpretation would be unwise, due to the large phase error bars, of order ± 40 degrees. Using the observed tide at Delfzijl as a reference, the longitude variation of the phase differences scatter around the variation expected of the equilibrium. From these two coherency phase results, it appears that a strong observational argument may be made in favor of an oceanic pole tide which progresses across northern Europe with the equilibrium tide. In this case, coherency amplitudes are much higher, averaging to .96 over all the stations at the pole tide frequency band, and .7 to .9 away from that frequency. Presumably, the high coherence estimate is due in part to coherent noise. The high coherency of the noise continuum is discussed in a later section.

Two sets of closely spaced stations give us an opportunity to monitor the detailed variation of the pole tide along a coast line. In the Gulf of Bothnia nine stations for the years 1924-64 span 1000 km. The whole gulf essentially goes up and down in unison at the pole tide frequency, with an average amplitude of 3.1 cm. The pole tide amplitude and phase difference (with respect to Kemi (I) at the northern end of the Gulf) information is collected in Table 7. The amplitudes, as plotted in Figure 8a, show no trend, and vary by about 20 per cent. The stations are located on the map according to the station index letters of Table 7. The phase difference (Figure 8b), does not deviate significantly from zero along the Gulf. Nedre Gavle (I) on the west has nearly the same amplitude as Mantyluoto (F) and Turku (G) on the east. The island observation at Degerby (H) in the mouth of the Gulf gives evidence for the lack of spatial variation of the tide in the interior of the basin.

The nine Dutch stations for 1900-64 span 300 km of coast line from Vlissingen (3) in the south to Delfzijl (11) in the north. The amplitudes are plotted in Figure 9a. At 439 days the amplitude grows by a factor of 2.5 across the nine stations, from .73 to 1.79 cm. The even larger amplitude, 2.75 cm at Esbjerg (12) on the western coast of Denmark, is consistent with this growth. The slope along the Dutch coast is approximately 1 cm/300 km. The phase difference information, referenced to the Helsinki record, is plotted in Figure 9b. Contrasts of 10-20 degrees appear, with the lower leading the upper part of the coast, but the error bars are too large to substantiate that suggestion.



Pole Tide Gulf of Bothnia 1924-64

Figure 8a

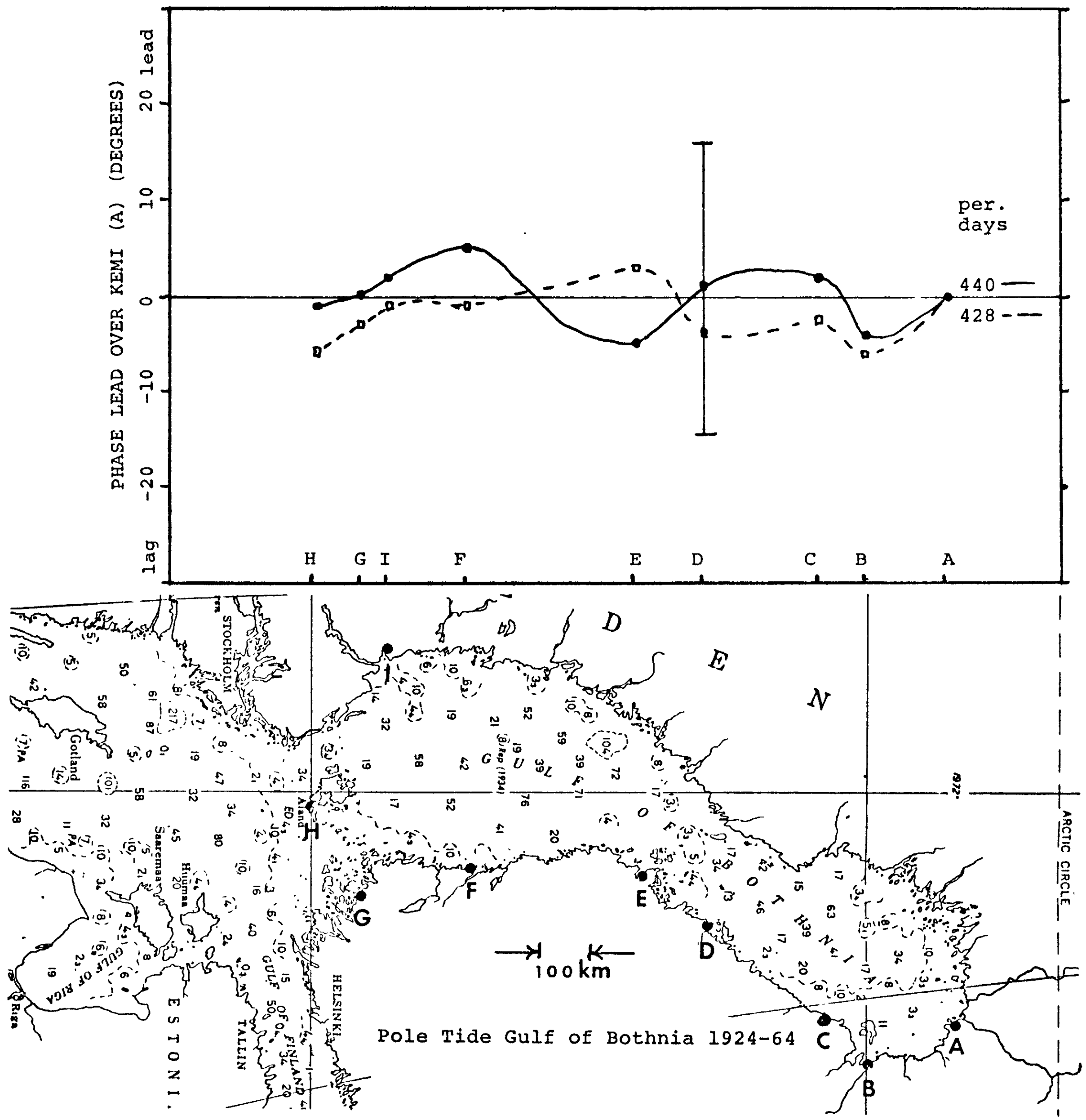
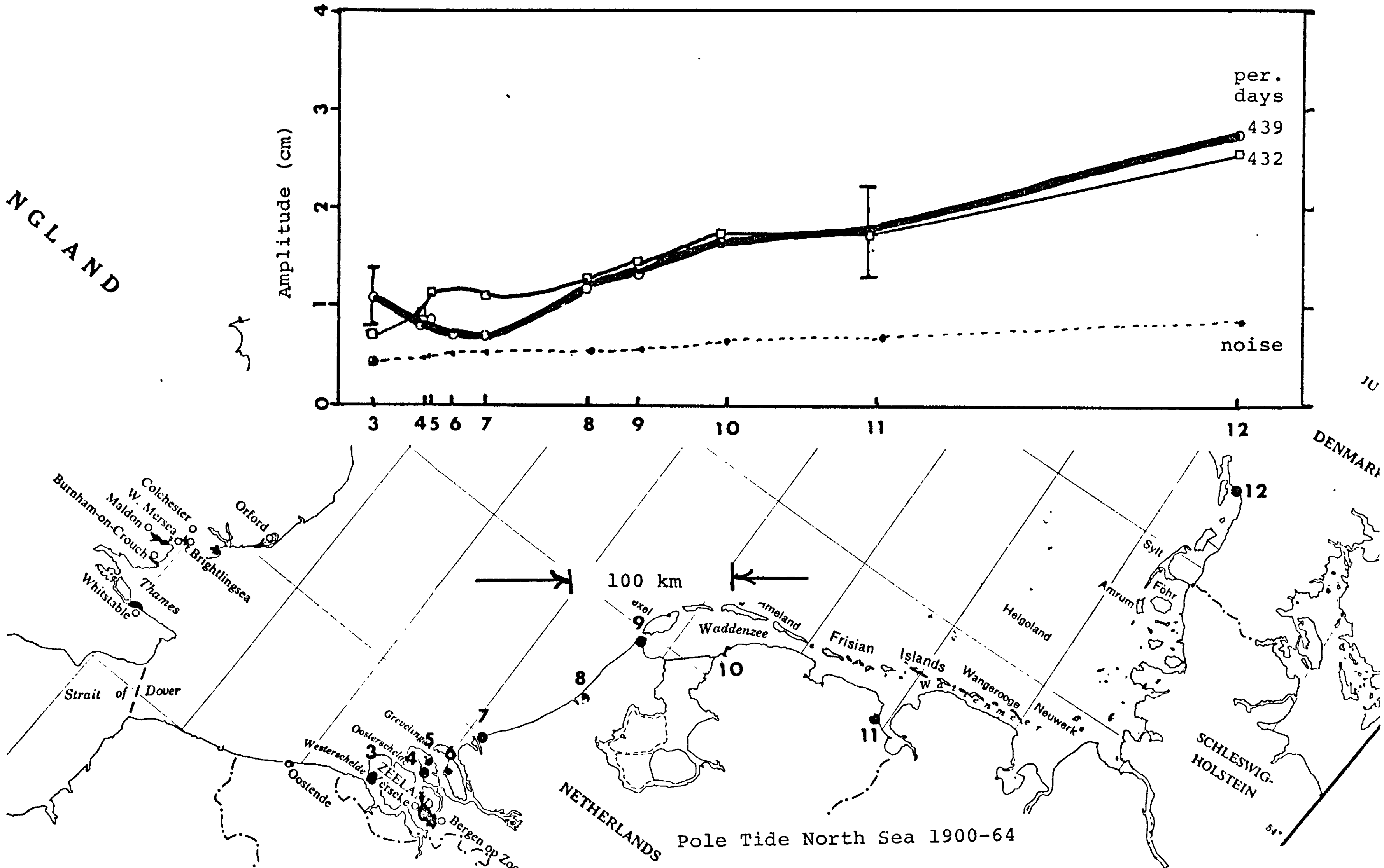

PHASE LEAD OVER KEMI (A) (DEGREES)
lag
lead
20
10
0
-10
-20
per.
days
440
428
H
G
I
F
E
D
C
B
A
STOCKHOLM
Gotland
Saaremaa
Hiiumaa
GULF OF RIGA
Riga
ESTONI.
TALLIN
GULF OF FINLAND
HELSINKI
Aland
100km
ARCTIC CIRCLE
Pole Tide Gulf of Bothnia 1924-64

Figure 8b



Pole Tide North Sea 1900-64

Figure 9a

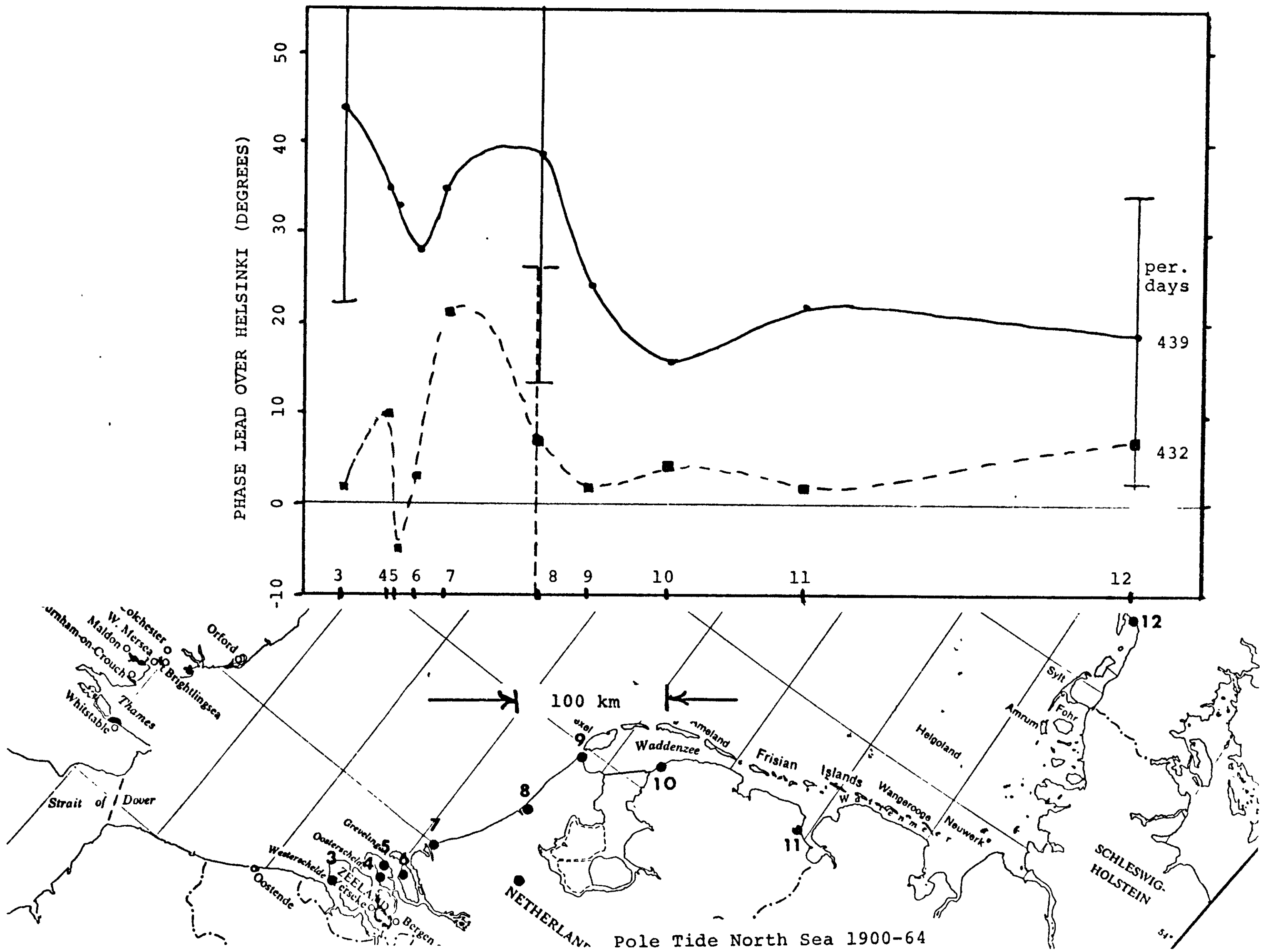

PHASE LEAD OVER HELSINKI (DEGREES)
50
40
30
20
10
0
-10
3
4 5
6
7
8
9
10
11
12
per.
days
439
432
100 km
Orford
Maldon
W. Mersea
Brightlingsea
Thames
Whitstable
Strait of Dover
Oostende
Westerschelde
Oosterschelde
Grevelingen
ZEELAND
Bergen
NETHERLANDS
Waddenzee
Frisian Islands
Wangerooge
Neuwerk
Helgoland
Amrum
Fohr
Sylt
SCHLESWIG-HOLSTEIN
Pole Tide North Sea 1900-64

Figure 9b

C. The Oceanic Context of the Pole Tide

1. The annual variations in sea level

Compared to the pole tide, the annual component of sea level is blessed with a large amplitude (6-10 cm) and a high signal to noise ratio (8-20). However, a precise hydrodynamic interpretation of the results is hindered by the lack of an adequate knowledge of the distributed sources (e.g., rainfall, run-off) that are important in the shallow seas of Northern Europe.

The amplitude of the annual component in sea level for Northern Europe for the period 1900-64, is plotted in Figure 10a as a function of latitude. The largest variation, with amplitude 10.2 cm and thus a range of 20.5 cm, is found at Esbjerg, on the shallow western coast of Denmark. In general though, there is an increase in amplitude from 6 cm in the North Sea to 9 cm in the Gulf of Bothnia. One should not be too hasty in interpreting the growth of amplitude in Figure 10a as a function of latitude, since the same data plotted as a function of longitude shows a similar growth. Local conditions predominate, rather than just a variation of over all forcing function. The phase information in Figure 10b tells us that the annual component is earliest in the shallow and land-surrounded waters around Zealand and latest on the western edge of the North Sea, in Great Britain. High water comes in the fall. In a local regime, such as the coast

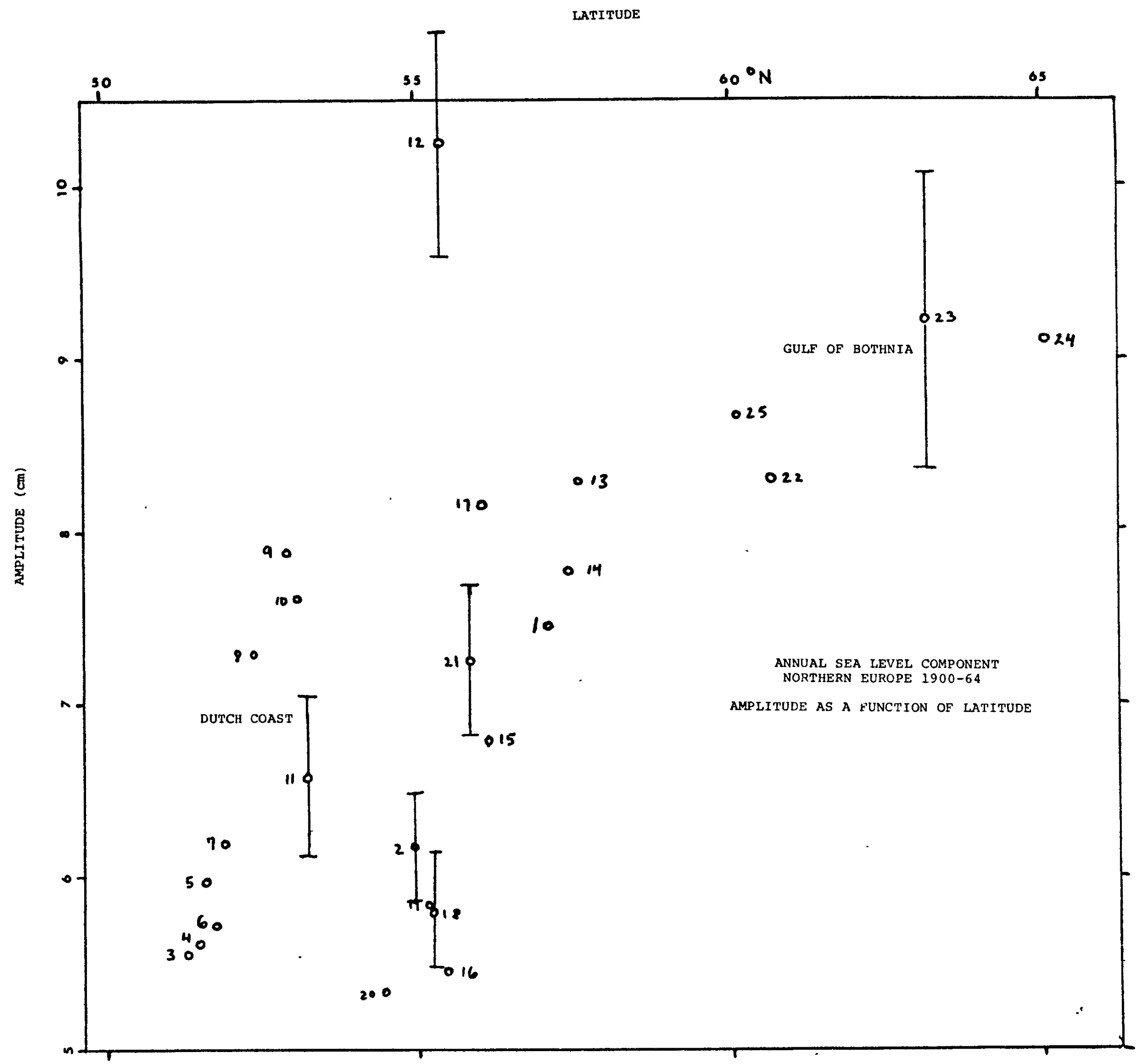

LATITUDE
50
55
60 °N
65
AMPLITUDE (cm)
10
9
8
7
6
5
GULF OF BOTHNIA
DUTCH COAST
ANNUAL SEA LEVEL COMPONENT
NORTHERN EUROPE 1900-64
AMPLITUDE AS A FUNCTION OF LATITUDE

Figure 10a

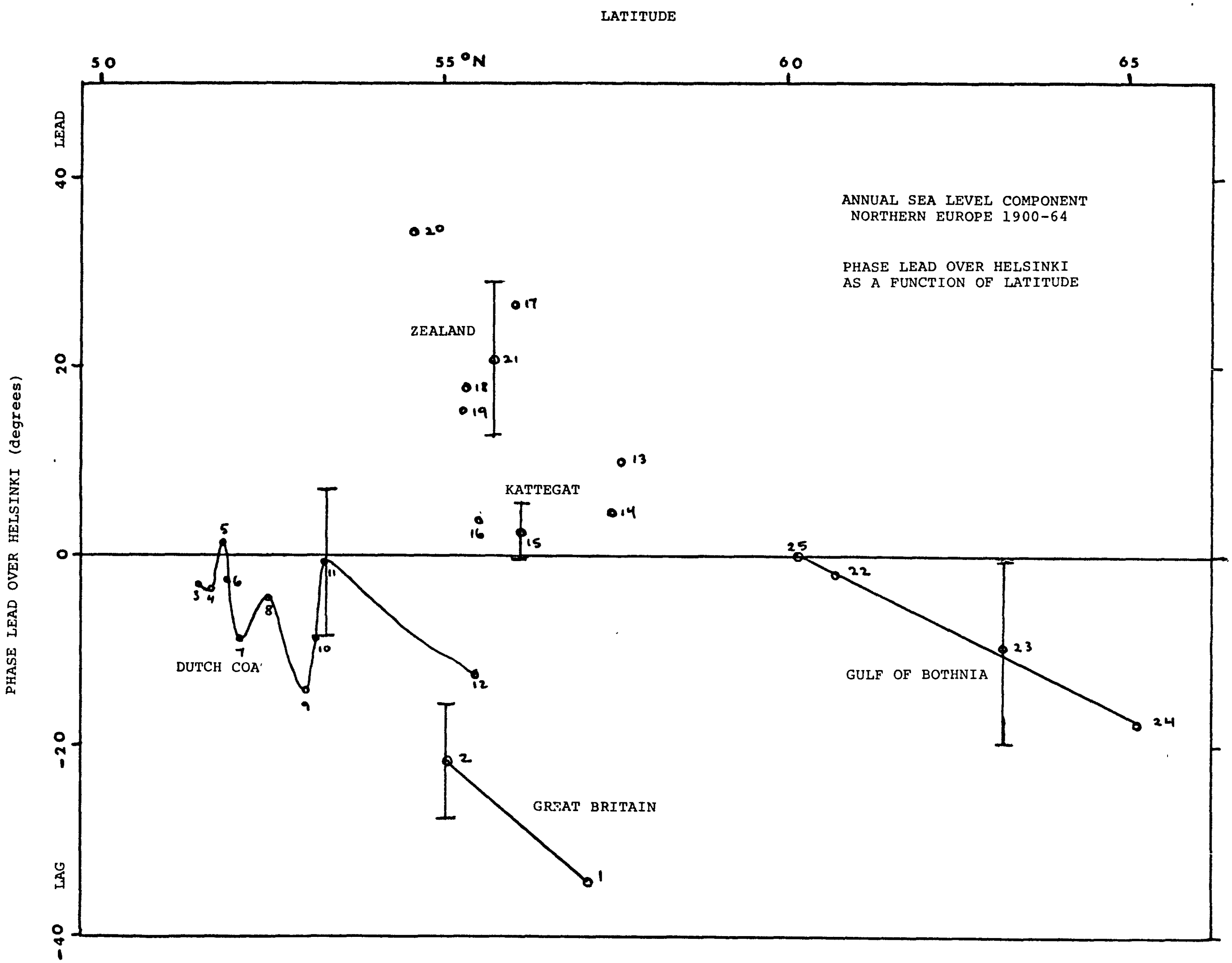

LATITUDE
50
55 °N
60
65
LEAD
LAG
PHASE LEAD OVER HELSINKI (degrees)
40
20
0
-20
-40
ANNUAL SEA LEVEL COMPONENT
NORTHERN EUROPE 1900-64
PHASE LEAD OVER HELSINKI
AS A FUNCTION OF LATITUDE
ZEALAND
KATTEGAT
DUTCH COA'
GULF OF BOTHNIA
GREAT BRITAIN

Figure 10b

of Great Britain, the Gulf of Bothnia, or the Zealand-Katlegat area, high water appears in the south earlier than in the North, by no more than about 20 days over 5 degrees latitude. The means of the same-named months over the period 1900-64 are plotted in Figure 11 for the earliest station, Gedser (20), the latest station Aberdeen (1), and for Helsinki (25). Significant departures from sinusoidal variation are in evidence. At Helsinki, pronounced annual peaks appear in the spectra of sea level (8.75 cm, maximum in August-September) atmospheric temperature (10.2 °C, maximum in July), and precipitation (1.5 cm, maximum in August), but not in atmospheric pressure (less than .1 mb). The reader is referred to Patullo, et al. (1955) for a detailed and comprehensive analysis of the world wide annual oscillation in sea level.

MEANS OF SAME-NAMED MONTHS, 1900-64

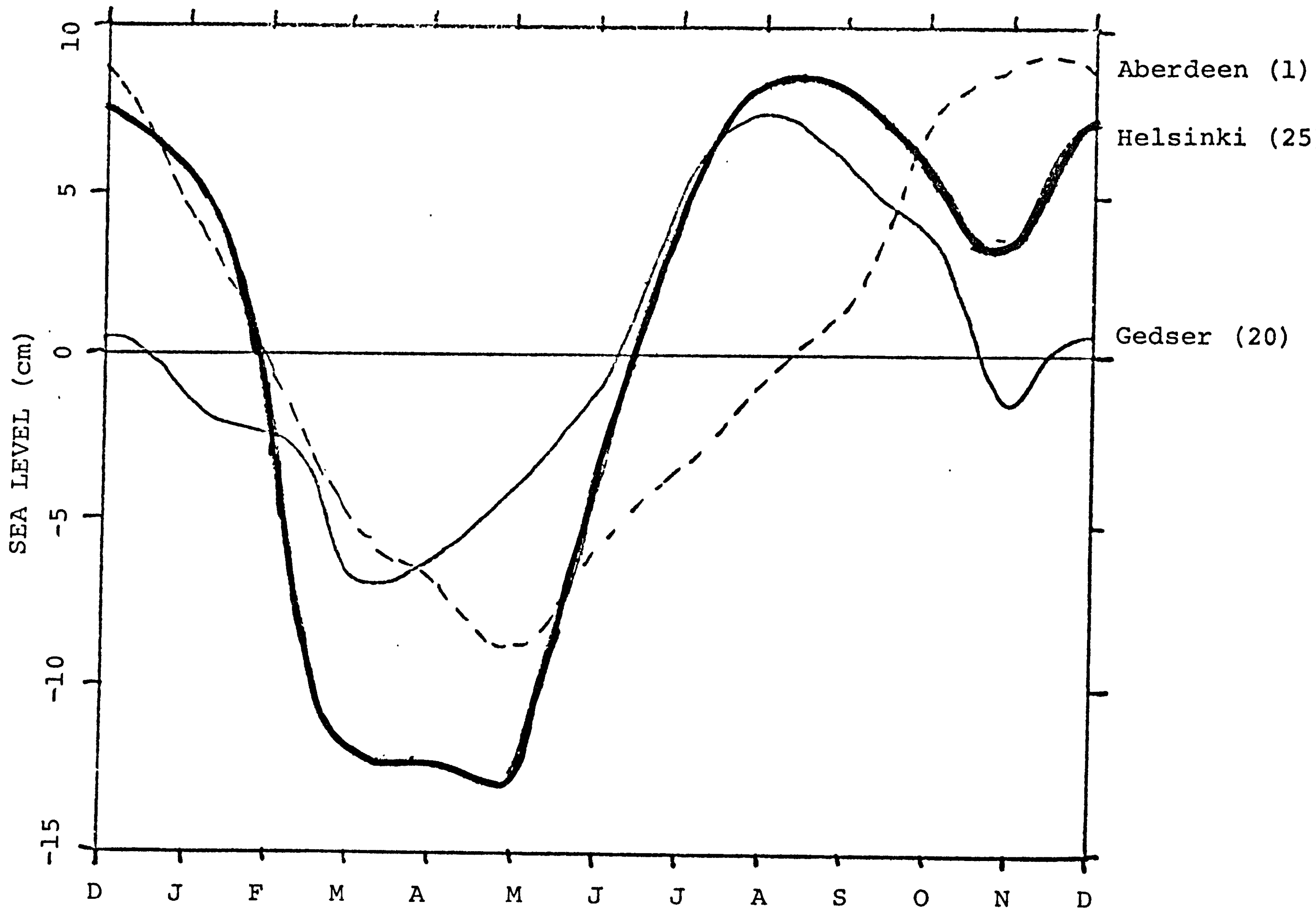


Figure 11

2. The Behavior of the Continuum

The similarity of the behavior of the background noise to that of the pole tide and the annual term motivates a further investigation of the sea level continuum. The Fourier transforms on monthly mean data, taken to study the pole tide, may give us information for periods DC to 2 months. In this application we may trade off frequency resolution to gain high statistical reliability, by using averages over 15 frequencies in computing power spectra and coherencies. Five representative spectra are plotted in Figure 12. The annual component and its harmonics were removed before transforming. The highest power is found at Nedre Gavle, in the Gulf of Bothnia. At frequencies below 3 cycles per year (4 months period), it has significantly more power than any other record. Furthermore, its spectrum has the most "structure" to it, in the sense of both more, and better-defined, peaks and troughs. Delfzijl, at the northern end of the Dutch coast, has about half the power of Nedre Gavle, at frequencies below 3 cpy. Above that frequency, the two are comparable. Vlissingen, at the southern end of the Dutch coast, generally has one tenth the power of Nedre Gavle. Cascais, on the coast of Portugal, facing the open Atlantic, has the least power of the sea level records, although it is comparable to Vlissingen above 3 cpy. In a hydrostatic situation, sea level and atmospheric pressures are related by an "inverse barometer" effect, so that an atmospheric pressure

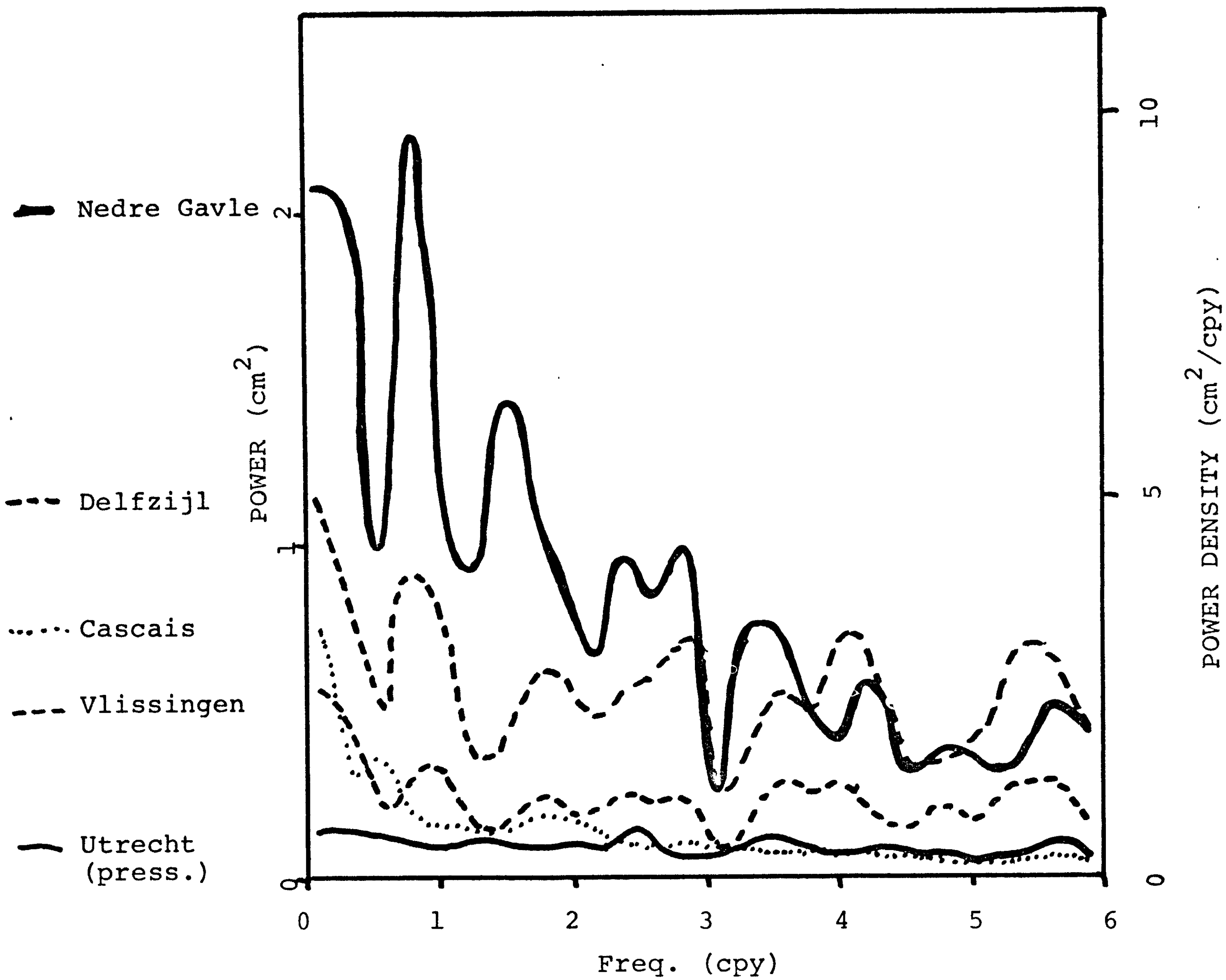

Nedre Gavle
Delfzijl
Cascais
Vlissingen
Utrecht (press.)
POWER (cm^2)
POWER DENSITY (cm^2/cpy)
0
1
2
0
5
10
0
1
2
3
4
5
6
Freq. (cpy)

Figure 12

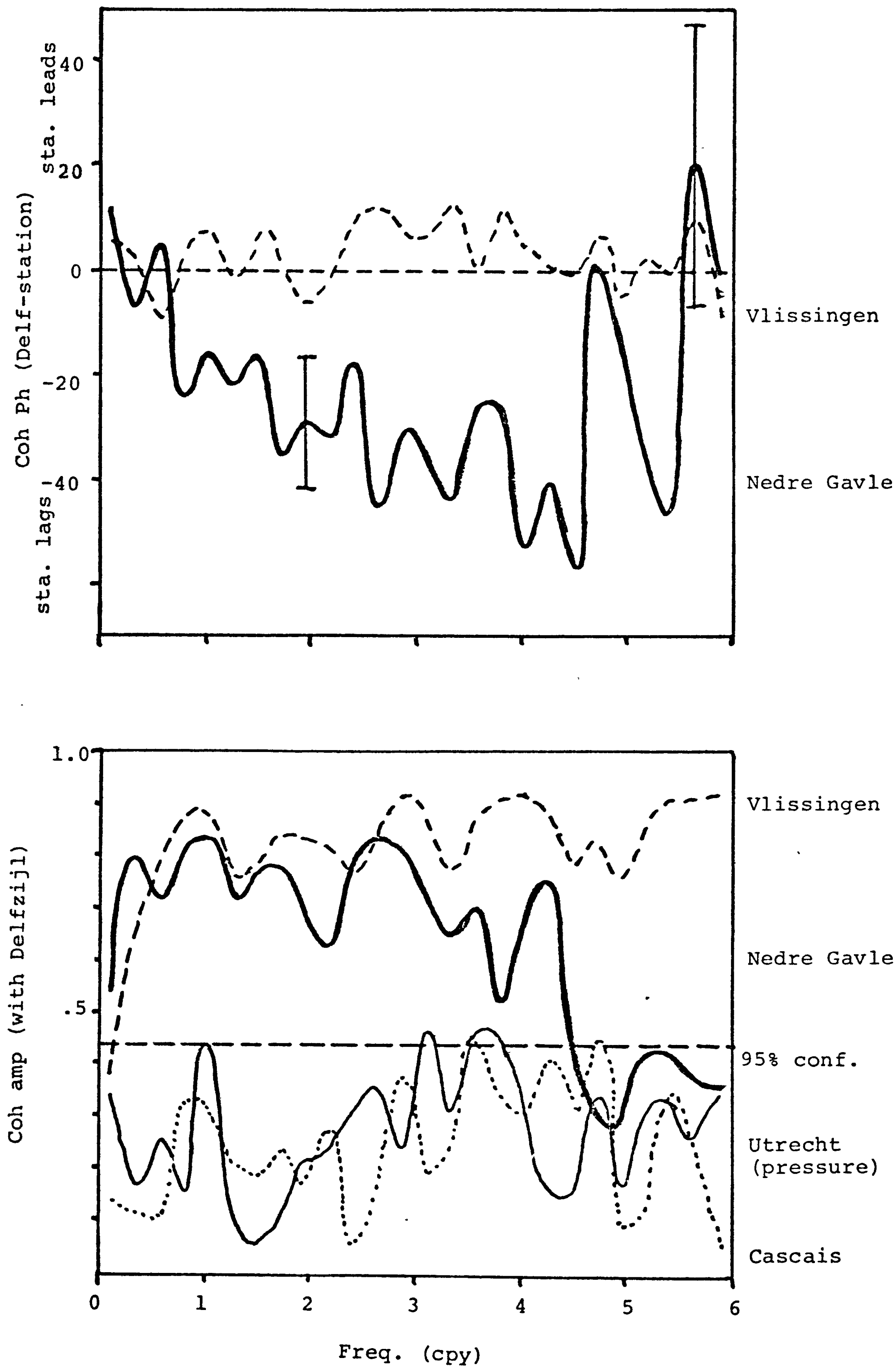
high of 1 mb would lower sea level by about 1 cm. The spectrum of atmospheric pressure from Utrecht in the Netherlands is also plotted, based on units of millibars. Below 3 cpy, its power is lower than any sea level record. Above that frequency, it is comparable only with Cascais. From this result one tends to conclude that local atmospheric pressure forcing of the sea level is too weak to account for the observed sea level continuum power in northern Europe.

We observe high noise power in northern Europe. The next question is, over what sort of length scales does the noise remain coherent? The coherency was computed between Delfzijl and each of the other stations represented in Figure 12, and plotted in Figure 13. The coherency with Cascais and with Utrecht pressure fails to rise above the significance level. Delfzijl is highly coherent with its neighbor Vlissingen, with a phase lag not significantly different from zero. Delfzijl is also highly coherent (.7 to .8) with Nedre Gavle, 1800 km away, in the Gulf of Bothnia, for periods longer than 80 days. The most striking feature of the coherency phase is the greater lag of Nedre Gavle behind Delfzijl, as the frequency increases from 0 to 4.5 cpy. Over this range, the phase is related to the frequency in a roughly linear fashion

$$\varphi = -\alpha \omega$$

where

$$\omega t + \varphi = \omega (t - t_0)$$



40
20
0
-20
-40
sta. leads
sta. lags
Coh Ph (Delf-station)
Vlissingen
Nedre Gavle
1.0
.5
Coh amp (with Delfzijl)
Vlissingen
Nedre Gavle
95% conf.
Utrecht (pressure)
Cascais
0
1
2
3
4
5
6
Freq. (cpy)

Figure 13

In this case, all the Fourier components have the same time origin t_o, as determined by the slope of the phase curve, $t_o = \alpha$. This suggests a group disturbance travelling to the east from the North Sea through the Baltic, with frequency content from 0 to 4.5 cpy, with travel time 10 days, and average speed 2 meters per second, or about 4.5 miles per hour.

In Figure 14 the coherence between the sea level records is explored further. The average amplitude of the coherencies over all the frequencies (0-6 cpy), between Helsinki and Copenhagen is very high, .89. Although almost 1000 km apart, these records are more coherent than Delfzijl and Vlissingen, only 300 km apart, with an average coherence of .82. Helsinki, 1835 km from Delfzijl by a winding path, is almost twice as coherent at frequencies below 3 cpy as Aberdeen with Delfzijl, only 675 km away on the coast of Great Britain. Although the phase error bars are larger for Aberdeen and Delfzijl, it is clear that the slope of the coherence phase is positive, whereas all the stations to the east of Delfzijl have negative slopes. Thus Aberdeen leads Delfzijl. If the group disturbance moves in the same direction as the phase velocity, this result implies that the source of the disturbance is not on the Dutch Coast, but further out towards the mouth of the North Sea, or perhaps the continental shelf. Most of the delay between Delfzijl and Helsinki occurs in the Kattegat and around Zealand. Copenhagen is close

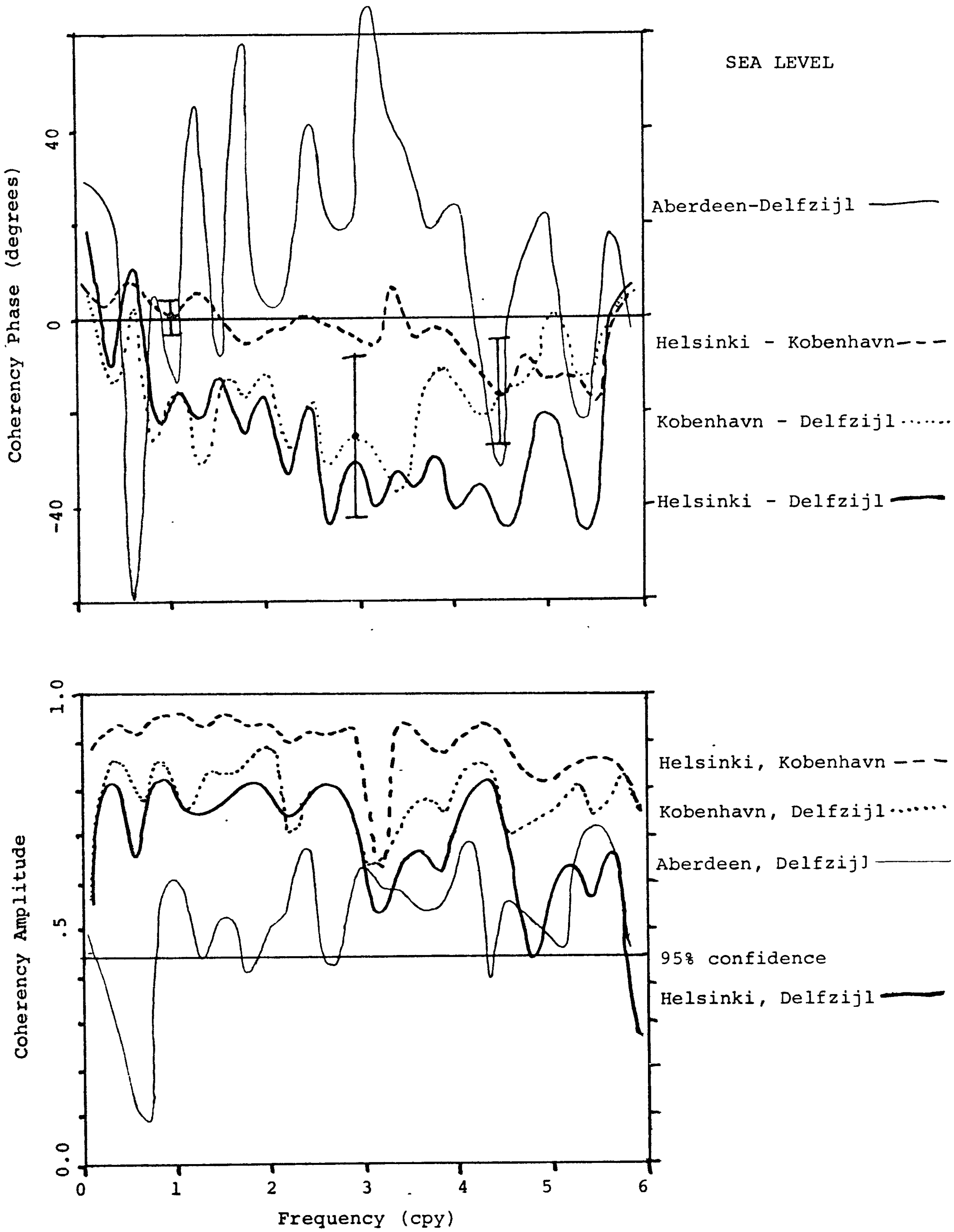

SEA LEVEL
Coherency Phase (degrees)
40
0
-40
Aberdeen-Delfzijl
Helsinki - Kobenhavn
Kobenhavn - Delfzijl
Helsinki - Delfzijl
Coherency Amplitude
1.0
.5
0.0
Helsinki, Kobenhavn
Kobenhavn, Delfzijl
Aberdeen, Delfzijl
95% confidence
Helsinki, Delfzijl
0
1
2
3
4
5
6
Frequency (cpy)

Figure 14

to half way between these two stations, yet the slope of the coherency phase curve with Delfzijl is only slightly less steep than that of Helsinki with Delfzijl. The travel time from Delfzijl to Helsinki, 8.5 ± 1.7 days, is slightly shorter than to Nedre Gavle, 10.1 ± .8 days, which is beyond the shallow and constricted waters of the mouth of the Gulf of Bothnia. Table 8 summarizes the deductions from the slopes of the coherency phases between selected stations. The distances are taken as the most direct sea route. In the Baltic, it is tempting to interpret the very rapid speed, 32.8 m/sec, as being due to gravity wave propagation. The gravity wave speed is 31 m/sec for the $h = 100$ m depth which is typical of the path of propagation. The speed between Delfzijl and Copenhagen is 1.5 m/sec, which is much too slow for a gravity wave.

TABLE 8

DEDUCTIONS FROM COHERENCY PHASE CURVES

			Distance	Travel Time	Speed	(range of speed)
Aberdeen	to	Delfzijl	675 km	10.1 ± 3.4 days	.77	(.58, 1.16) m/sec
Delfzijl	to	Copenhagen	870	6.8 ± 1.7	1.49	(1.19, 1.98)
Delfzijl	to	Nedre Gavle	1800	10.1 ± .8	2.06	(1.90, 2.25)
Delfzijl	to	Helsinki	1835	8.5 ± 1.7	2.61	(2.16, 3.28)
Copenhagen	to	Helsinki	965	3.4 ± .8	32.8	(26.3, 43.6)

3. Sea Level and Weather

Seeking an explanation for the behavior of the continuum of sea level in terms of local atmospheric forcing, a preliminary analysis of weather data from northern Europe has been undertaken. In the North Sea, at Aberdeen (1) and Vlissingen (3), sea level and atmospheric pressure are significantly coherent, and 180 degrees out of phase, within the error bars. See Figure 15a. This is the expected hydrostatic, inverse barometer effect. However, at the northern end of the Dutch coast, Delfzijl sea level coherence with pressure is below the level of significance, the coherency amplitude being only about half as great as at Vlissingen. The pressure record is from Utrecht, 50 km inland and roughly equidistant from both tide stations.

At Copenhagen (21) in the constricted waters at the western end of the Baltic Sea, sea level and pressure are only marginally coherent, as shown in Figure 15b. The error bars, about $\pm$ 30 degrees, all include the 180 degree phase lag, but in the neighborhood of 2-4 cpy sea level lags behind pressure by about 210 degrees. At Helsinki (25) on the eastern end of the Baltic, sea level and pressure are significantly coherent. The phase lag of sea level behind pressure is 180 degrees at the high and low frequency limits, but not in between. The lag grows to about 230 degrees at 3.5 cpy, where the error bars do not include 180 degrees. This represents a lag of 14 days behind the inverse barometer

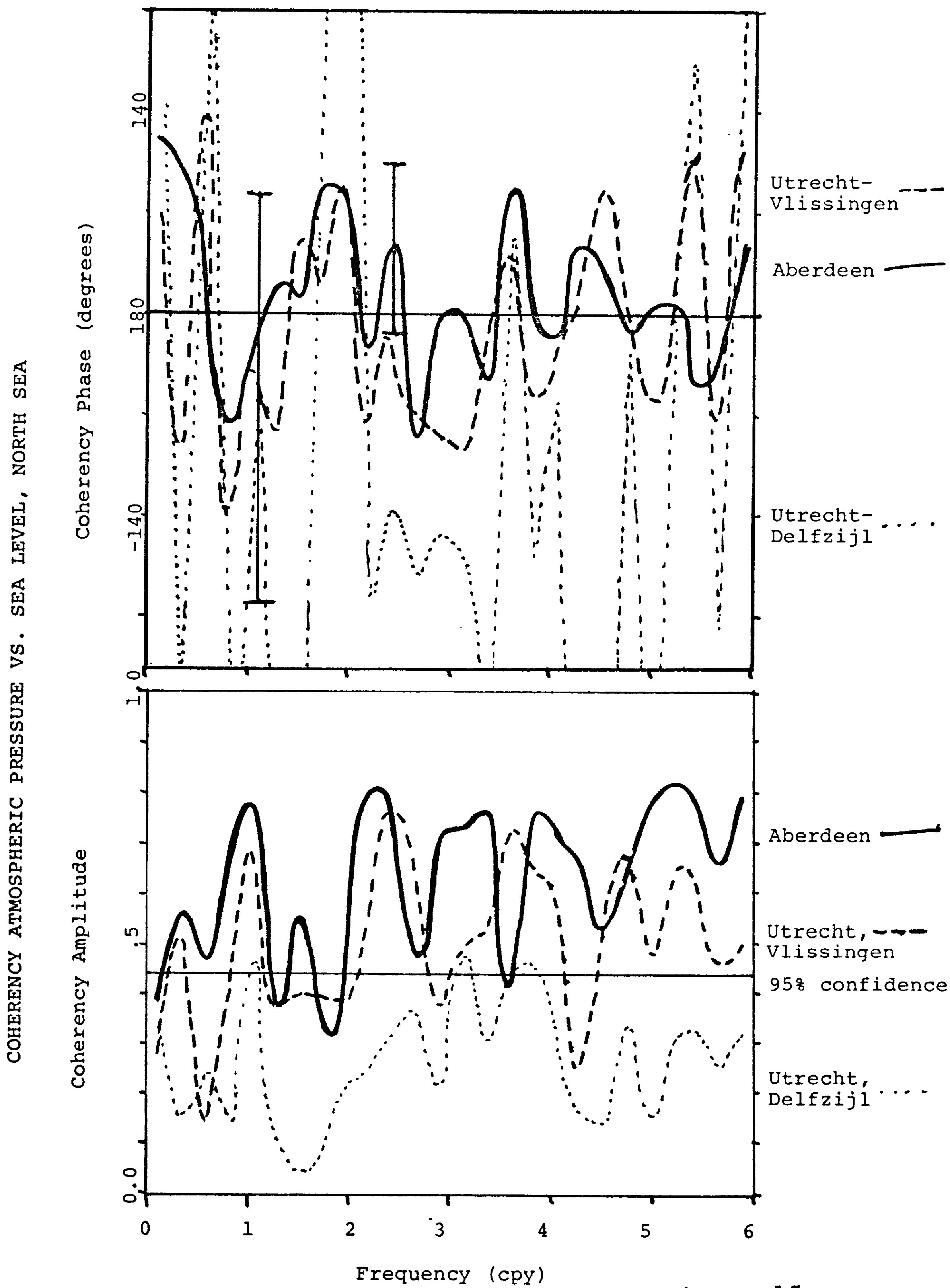

COHERENCY ATMOSPHERIC PRESSURE VS. SEA LEVEL, NORTH SEA
Coherency Phase (degrees)
140
180
-140
0
Utrecht-
Vlissingen
Aberdeen
Utrecht-
Delfzijl
Coherency Amplitude
1
.5
0.0
Aberdeen
Utrecht,
Vlissingen
95% confidence
Utrecht,
Delfzijl
0
1
2
3
4
5
6
Frequency (cpy)

Figure 15a

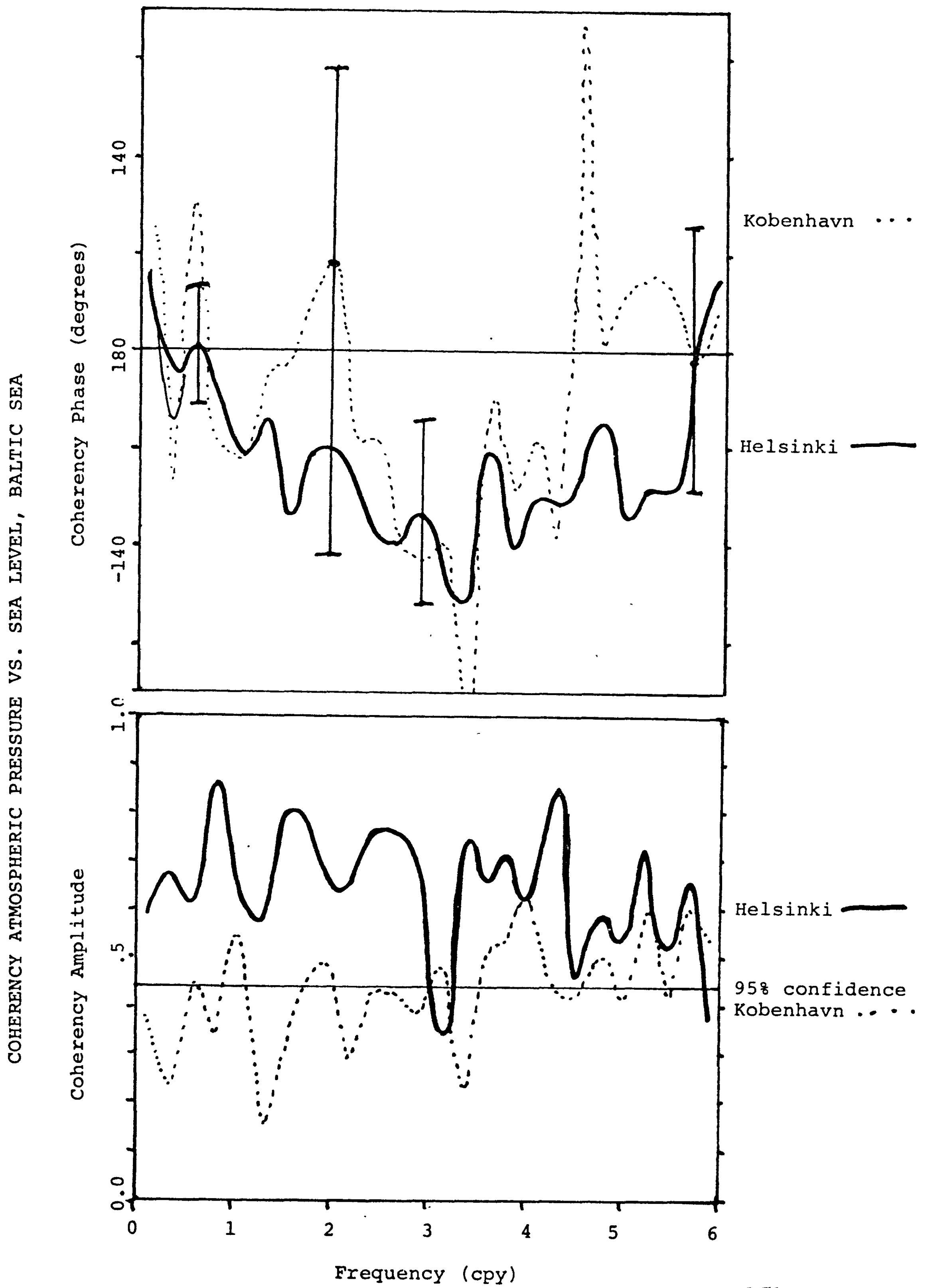

COHERENCY ATMOSPHERIC PRESSURE VS. SEA LEVEL, BALTIC SEA
Coherency Phase (degrees)
140
180
-140
Kobenhavn
Helsinki
Coherency Amplitude
1.0
.5
0.0
Helsinki
95% confidence
Kobenhavn
0
1
2
3
4
5
6
Frequency (cpy)

Figure 15b

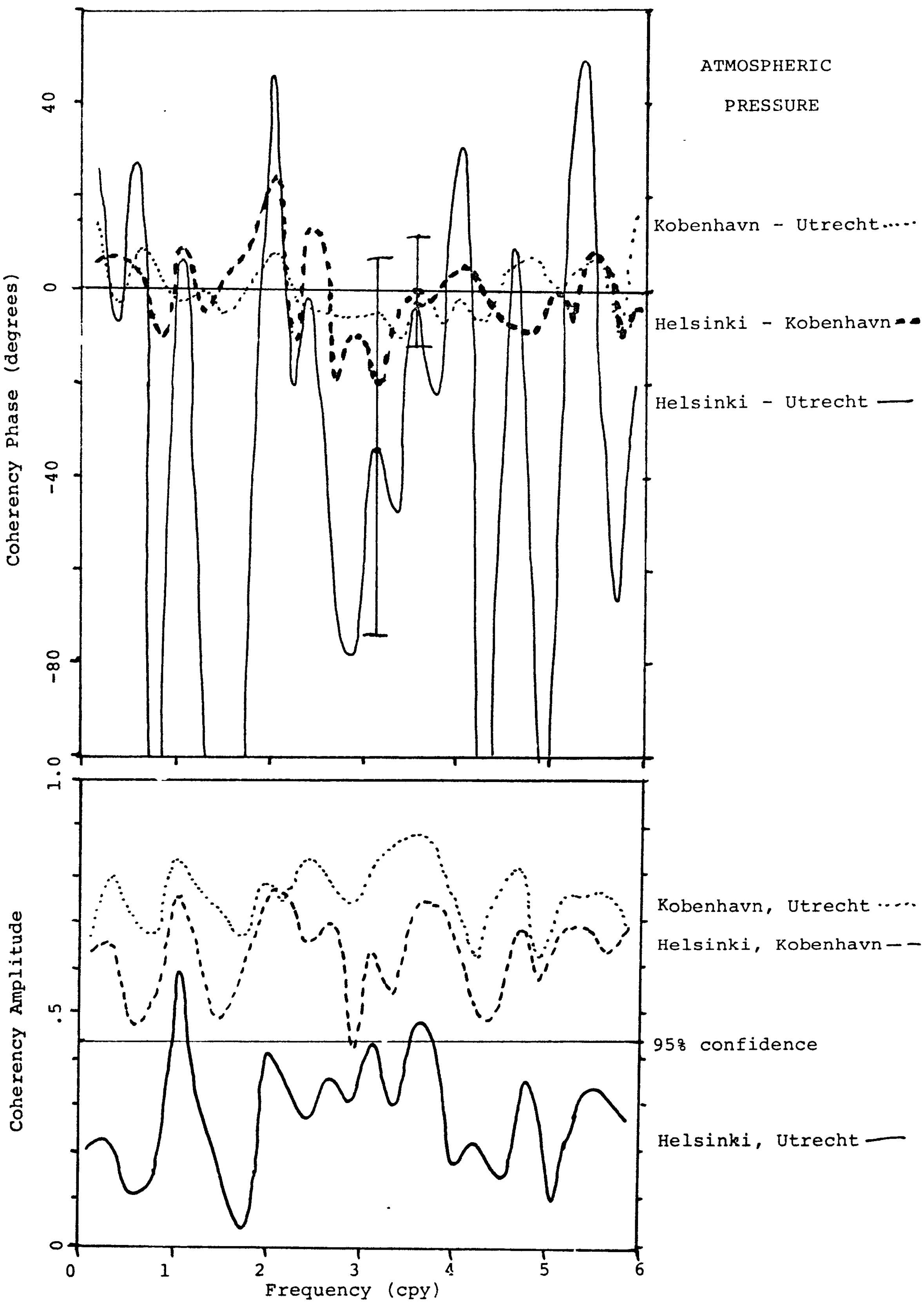

ATMOSPHERIC
PRESSURE
Kobenhavn - Utrecht
Helsinki - Kobenhavn
Helsinki - Utrecht
Coherency Phase (degrees)
40
0
-40
-80
1.0
.5
0
Coherency Amplitude
Kobenhavn, Utrecht
Helsinki, Kobenhavn
95% confidence
Helsinki, Utrecht
0
1
2
3
4
5
6
Frequency (cpy)

Figure 16

relationship, for a component with period 104 days.

The pressure records at Copenhagen and Utrecht are quite coherent with each other, and do not deviate significantly or systematically away from zero phase lag. That is also the case between Copenhagen and Helsinki. However, Helsinki and Utrecht are not significantly coherent and the phase lag oscillates widely. The coherencies are plotted in Figure 16. Thus we see that sea level in northern Europe is coherent over much greater length scales than is atmospheric pressure.

The other tabulated monthly mean weather variables at Helsinki, precipitation and atmospheric temperature, are much less coherent than is pressure with sea level. See Figure 17. Marginally significant coherence exists only in the region 3-4 cpy. The phase lags oscillate widely about zero. As a matter of fact, the coherency of atmospheric temperature with sea level may be only an indirect effect, in which temperature influences sea level only through the temperature dependence of pressure. The influence of wind has been left out, only because of a lack of data. The effect is probably quite important, as noted in the section Discussion of the Behavior of the Continuum, below. A complete study requires a multiple regression of all the weather variables on sea level, in the manner of Wunsch (1972), for example.

We have seen that sea level need not be coherent with local atmospheric pressure, that pressure coherencies break down at distances of order 1000 km, while sea level coherencies extend twice that far, and also that pressure coherencies do not exhibit the sloping coherency phase, which was so characteristic of the sea level coherency phase behavior.

COHERENCY WEATHER VS. SEA LEVEL, HELSINKI

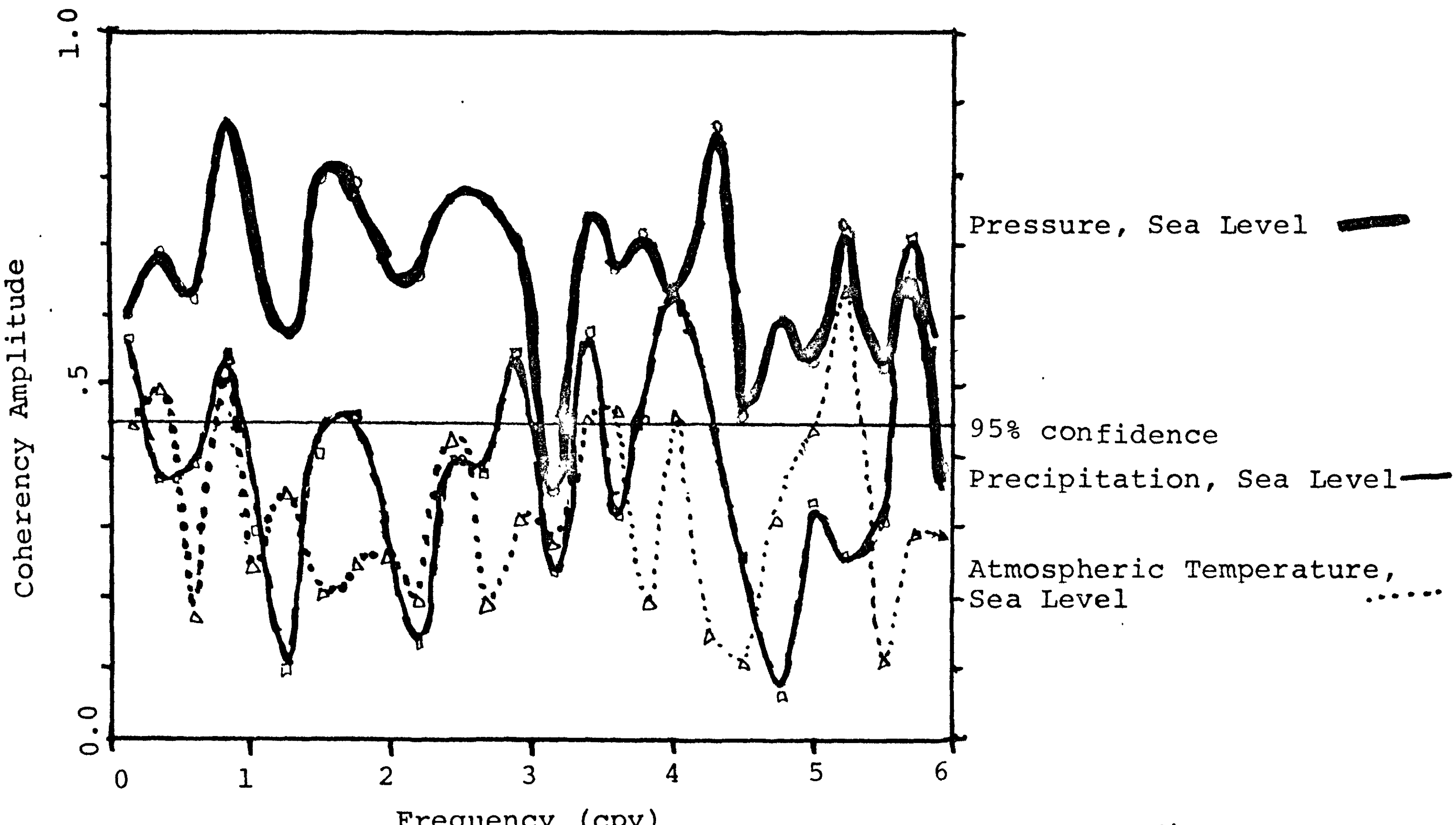


Figure 17

PART II. INTERPRETATION AND DISCUSSION

A. Statement of the Problems Raised By the Observations

We shall concentrate on two problem areas, the pole tide and the low frequency sea level continuum, as posed by the preceeding long term observations of sea level in northern Europe. From the original motivation, we would like to understand how pole tide amplitudes greater than equilibrium might be generated. Large amplitudes in the North Sea, and even larger amplitudes in the Baltic require an explanation. Furthermore, a variation appears along the Dutch coast, with a rise in pole tide amplitude of 1 cm over 300 km. As regards the continuum,one would like an explanation for the observed growth of power, in the rough ratio 1:5:10 from the Atlantic and lower Dutch coast, to the upper Dutch coast, and into the Baltic. One would also like an understanding of the curious, highly coherent behavior of the sea level continuum, with frequency content 0-4 cycles per year, and slow phase propagation to the east, 2 m/sec, in the North Sea and through the Kattegat, and a higher speed, 30 m/sec, across the Baltic.

B. The Problem of Variation Along Coast Lines

Two closely spaced arrays of tide stations have given us two different results of the behavior of the pole tide along coast lines. The pole tide is constant along the coast of the Gulf of Bothnia, but appears to vary along the Dutch coast. What does the dynamical theory have to say on this matter?

Let us derive the boundary conditions on sea level, in a rotating basin, for vanishing normal flow into a coast line. Following Lamb (1945), section 207, the inviscid, linearized, vertically integrated conservation of horizontal momentum equations give us

$$U_t - f V = -g \zeta'_x$$

$$V_t + f U = -g \zeta'_y$$

where the current U is in the x (east) direction, and V is in the y (north) direction. The rotation is characterized by the coriolis parameter, $f = 2\Omega \sin \theta$. The dynamic part of the tide is $\zeta' = \zeta - \bar{\zeta}$, the difference of the observed tide ζ and the equilibrium tide $\bar{\zeta}$. Assuming an $e^{i\sigma t}$ time dependence, we may solve for the currents

$$U = \frac{g}{f} \frac{1}{1-(\sigma/f)^2} \left\{ i\frac{\sigma}{f} \zeta'_x - \zeta'_y \right\}$$

$$V = \frac{g}{f} \frac{1}{1-(\sigma/f)^2} \left\{ \zeta'_x + i\frac{\sigma}{f} \zeta'_y \right\}$$

The boundary condition is no normal flow through the walls of the basin. For example, at north-south boundary, the east-west current must vanish. That is

$$\frac{i\sigma}{f}\zeta'_x - \zeta'_y = 0$$

If the length scales in the x and y directions are L_x and L_y, respectively, the boundary condition involves the terms

$$\frac{\zeta'}{L_y}\left(\frac{i\sigma}{f}\frac{L_y}{L_x} - 1\right) = 0$$

There are 2 regimes of solution. First, we may consider $\frac{\sigma}{f}\frac{L_y}{L_x} \doteq 1$. That is, sea level gradients normal to the coast may balance the sea level gradient tangential to the coast. The other regime is $\zeta'/L_y = 0$. This is satisfied either by $\zeta' = 0$, no dynamic tide at all, or $L_y = 0$, no variation along the coast line.

The first regime applies to ordinary "high frequency" tidal problems, but at the pole tide frequency, for latitude 55 degrees (North Sea) $\sigma/f = 1/710$. This means that variations normal to the coast would need slopes 710 times greater than those for variations tangential to the coast. The observed pole tide tangential slope of 1 cm/300 km would imply a normal slope of 7.1 meters/300 km in the interior of the basin. This is an absurdly large slope, which would in turn imply an absurdly large, and non-observed, geostrophic current, of order 2 meters/second, tangential to the coast. A realistic matching of this behavior for two coast lines making a curve at 90 degrees appears to be an impossible task.

Before completely rejecting the first regime, let us examine the influence of dissipation on the boundary condition, in the form of bottom friction. Following Proudman (1960) and Bowden (1953), we may consider the bottom stress to be of the form

$$\tau = .0025\rho\,|\vec{U} + \vec{v}|\,(\vec{U} + \vec{v})$$

where $\vec{U}$ is the large "high frequency" current of the M_2 tide, and $\vec{v}$ is the current associated with the pole tide. The linearized bottom stress is of the form $R\,\vec{v}$, where

$$R = \frac{1}{100\pi}\,\frac{|\vec{U}|}{h}$$

With $e^{-i\sigma t}$ time dependence, the introduction of the linearized bottom stress gives a decay with time e^{-Rt} , and we may define a complex frequency $\omega = \sigma - iR$. The boundary condition is now

$$\left(\frac{i\sigma}{f} + \frac{R}{f}\right)\zeta'_x - \zeta'_y = 0$$

With enough friction, the factor R/f may allow more reasonable normal gradients to balance tangential gradients of sea level along the coast line. From observations at several Dutch light ships over the period 1910-39 (Otto, 1964), we may take 60 $\pm$ 10 cm/sec to be a typical M_2 tidal current magnitude, at depths of 25 $\pm$ 5 m. This yields $R = .77 \pm .30 \times 10^{-4}$ sec^{-1}, or $R/f = .65 \pm .25$, as compared to $\sigma/f = 1/710$ for the pole tide. The damping time $1/R$ is only about 3.5 hours. If the pole tide and normal tides do interact in this fashion, this result suggests an extremely over damped case, $R/\sigma \gg 1$. This would allow reasonable sea level slopes normal to the coast to balance the observed slope along the coast, but it is difficult to understand how a highly damped solution could indeed be observed at all. In fact, the observed sharpness of the spectral peak of the pole tide requires a very weakly damped behavior.

We have been discussing a boundary condition which is no flow through the coast line. Suppose that we relax this constraint. What are the consequences? For an inviscid ocean, the geostrophic current required to balance the 1 cm/300 km slope in sea level along the coast line would be $v = -\frac{g}{f}\zeta'_y$ = -.29 cm/sec through the coast line. This is a tiny current, but when it flows for half a pole tide period, a tremendous mass of water is transported. Consider an idealized river, with a mouth of depth d, width w, and length l. For the pole tide current, $v_0 \sin \sigma t$, the volume of water entering it during a half period is $V = wd\int_0^{\pi} v_0 \sin \sigma t \; dt = 2wd\frac{v_0}{\sigma}$. The change in height of the river would be

$$\Delta\zeta = V/wl = 2\frac{v_0}{\sigma}\frac{d}{l}$$

For a river with d = 10 m, l = 100 km, v_0 = .29 cm/sec, at the frequency $\sigma = 1.67 \times 10^{-7}$ sec, the change in level would be enormous, $\Delta\zeta$ = 12 meters. The lower portion of the Dutch coast is primarily river delta, but it would take a hundred rivers to support a geostrophic pole tide current which would escape entry into farmer's almanacs. Thus we conclude that a leaky boundary cannot account for the observed slope in the pole tide along the Dutch coast.

It appears that the first regime of behavior, in which interior gradients balance slopes along the boundary, must be rejected. Since the pole tide is clearly enhanced over equilibrium, the preferred result from the second regime is that there should be no variation along a coast line. That is, sea level may vary harmonically with time, with amplitude equal to the equilibrium plus a constant along the boundary. Why the observed pole tide on the Dutch coast departs from this theoretical result must be considered to be a yet unanswered question.

C. Small-scale Resonance

1. A Small-scale Resonance Hypothesis

Part of the original intent of this project was to examine the possibility of resonance in ocean basins at low frequency. Clearly, such a resonance cannot be due to ordinary gravity waves, which travel at the high speed $\sqrt{gh}$. Such waves would cross the North Sea in less than a day, not fourteen months. However, when the variation with latitude of the rotation of the earth is taken into account, small length scales may emerge. For example, in Longuet-Higgins (1965) we find the dispersion relation for free Rossby waves

$$\sigma = \frac{-\beta k}{k^2+l^2+f^2/gh}$$

where β represents the variation of the coriolis parameter, $f = 2\Omega \sin\theta$, with latitude in a cartesian coordinate system, with $f = f_o + \beta y$, so $\beta = f_o \cos\theta / R_e$. The depth, h, is a constant. The wave numbers (k, l) are in the directions (east, north). One notes that this dispersion relation is anomalous. That is, low frequencies are associated with large wave number, i.e., short scales of motion.

The observations of Wunsch (1967) show that the long period tides Mf (13.66 days) and Mm (27.54 days) in the Pacific do not follow the equilibrium law. Rather, their spatial structure bears some consistency with the idea that the long period tidal forcing further excites the Rossby wave modes of the Pacific Ocean. The detailed comparison of theoretical dynamical to observed tides left some questions unanswered, but the result provided motivation for an examination of the dynamical aspects of the pole tide. Free Rossby waves at the pole tide frequency in mid-latitudes may have short wavelengths, of order 100 km, and slow phase speeds, of order .3 cm/sec. At such short length scales, one might think that local variations in topography would become quite important. In deriving an equation of motion, we find the terms β/f are accompanied by terms $\nabla h/h$. The reader may refer to Rhines (1969) for a treatment of "topographic-Rossby wavws." In the North Sea, $\nabla h/h \sim 1/400$ km is typical, while $\beta/f \simeq \cos\Theta_0/R_e \cong .57/6400$ km. Thus we would expect topographic gradients to actually dominate the β effect.

In principle, one could construct a finite difference compution model of the shallow seas of Northern Europe, including a hierarchy of physical effects, such as simple to complicated bottom topography, mean and transient currents, bottom friction or eddy viscosity, and variable boundary shapes. To a large degree, however, the results of this task can be predicted or rejected in advance. The rejection of the resonance amplification hypothesis will be examined in the following section.

2. Observational Arguments Against Small-scale Resonance

There are three features of the observations which contradict a small-scale resonance hypothesis. First, there is simply no small scale variation of amplitude observed. There are no modes, or phase differences greater than about 60 degrees evident throughout the observations in Northern Europe. One might say that this smooth result is to be expected on the basis of the low frequency limit of the boundary conditions, but that the response in the interior may well be quite structured. The second feature counters that hypothesis. An island observation, at Degerby (H) in the Gulf of Bothnia, differs from its neighboring stations on both sides of the gulf by less than 10 per cent in amplitude, and less than 3 degrees in phase.

The third and most convincing argument is the narrowness of the observed pole tide spectral peak. If small scales were involved, transient events, and interactions with mean currents would act to smear out the pole tide spectrum over a band of frequencies much wider than observed. To illustrate this point, consider the example of a wave, normally with frequency σ and wave number k , propagating now in a medium with a mean current U . To a fixed observer, e.g., to a tide gauge on the shore, the frequency is Doppler shifted to $\sigma - kU$. Mean currents of order 1 to 10 cm/sec are reported (Otto (1964)) along the Dutch coast, but the observed spectrum is not shifted or spread in frequency more than $2\Delta f$ from the observed Chandler wobble frequency band, where $\Delta f = 1/65$ years. This gives us an upper bound for the wave number, corresponding to a wavelength greater than 10^5 km, an enormously large scale.

With a small-scale phenomenon, one may expect non-linear effects, involving such terms as uu_x in the momentum balance. In that case, variations at twice the Chandler frequency would be introduced into the system. No peak appears above the noise level at twice the Chandler frequency, or at the sum or difference of the Chandler and annual frequencies. Although this observation does not completely rule out a linear, or weakly non-linear resonance, it lends the small-scale resonance hypothesis no strength.

D. Forcing

1. Direct tidal forcing of a circular basin

Sea level in a basin may be forced directly, by the tidal potential, or indirectly by the influence of an adjoining body of water. The latter, "co-oscillating" tide involves the response of a basin to a specified amplitude or current along a portion of its boundary. What forcing mechanism may be important for the pole tide? In this section, and the one to follow, this question is posed in the context of a circular basin of radius r_0, with a flat bottom of depth h, rotating with constant f. Consequently, we are dealing with large scales, rather than the small scales which would be typical of a Rossby wave response.

Following Lamb (1945), section 211, we have the governing equation for the observed tide $\zeta = \zeta' + \bar{\zeta}$

$$\nabla_1^2 \zeta - K^2 \zeta = \nabla_1^2 \bar{\zeta}$$

where $$\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

and $$K^2 = (f^2 - \sigma^2)/gh$$

We may consider a forcing function of the form

$$\bar{\zeta} = F e^{i\theta}$$

which has the property $\nabla_1^2 \bar{\zeta} = 0$. This corresponds to a plane tilting of the geoid, which models the local behavior of the equilibrium tide for a small basin. For low frequency, $\sigma < f$, the solution is related to the modified Bessel function of the first kind of order 1,

$$\zeta = A\, I_1(Kr)e^{i\theta}$$

Substitution into the boundary condition on the dynamic sea level

$$\frac{i\sigma}{f}\left(\zeta-\bar{\zeta}\right)_r - \frac{1}{r}\left(\zeta-\bar{\zeta}\right)_\theta = 0 \quad \text{on } r=r_0$$

gives the amplitude

$$A = \frac{-F\,(1-\sigma/f)}{\frac{\sigma}{f}\left\{Kr_0\, I_0\,(Kr_0) - I_1\,(Kr_0)\right\} - I_1\,(Kr_0)}$$

The solution does indeed achieve its maximum on the boundary, but the amplitude will be very close to equilibrium. Applied to the Baltic Sea in a rough fashion, h = 100 m, and θ = 60 degrees, so we have K = 1/265 km. Thus the case $Kr_0 = 1$ is a reasonable value. With $\sigma/f = 1/710$, we find that on the boundary the amplitude is less than one part in a thousand greater than equilibrium.

Intuitively, one would think that gradients of the pole tide forcing function would be extremely small over a small basin, since the scale of the equilibrium is of order the radius of the earth. The above result suggests that direct pole tide forcing of small basins is an equilibrium process.

2. Forcing of a basin at the boundary

From the strong tendency for sea level to be constant along coast lines, due to the boundary condition at low frequency, one would expect that a basin, co-oscillating with a tide of amplitude ζ_o at its mouth, would also oscillate with amplitude ζ_o all along its coast lines.

If an oscillating current u_0 is applied at the mouth, the flow of water in and out of the basin will cause a rise and fall in sea level. Again, from the boundary condition at low frequency, we would expect the sea level to be uniform around the coast lines. The amplitude of the oscillation in sea level will depend on the amount of flow, and the area of the basin.

Thinking of the pole tide in the Baltic Sea, which is connected by a narrow inlet to the North Sea, we may pose and solve an explicit problem. Again following Lamb (1945), the solution is of the form

$$\zeta = \sum a_s I_s (Kr) e^{is\theta}$$

where $K^2 = f^2/gh$. The boundary condition is vanishing radial current, excpet through an inlet of width w, in which the current is specified to be u_0.

$$u = \frac{g}{f} \left\{ \frac{i\sigma}{f} \zeta_r - \frac{1}{r_o} \zeta_\theta \right\} = \begin{cases} o \text{ for } & |\theta| > w/2r_o \\ u_0 \text{ for } & |\theta| < w/2r_o \end{cases}$$

Using a delta function times a constant, A $\delta(\theta)$, we may approximate the boundary condition for a small inlet. We have

$$\int_0^{2\pi} u \, d\theta = \int_0^{2\pi} A\,\delta(\theta)\, d\theta = A$$

The integral of the actual current is $u_0 \Delta\theta = u_0 w/r_0 = A$. Recalling $\delta(\theta) = \sum e^{is\theta}$, the boundary condition is

$$\sum a_s e^{is\theta} \frac{g}{f}\left\{ \frac{i\sigma}{f} \frac{\partial}{\partial r} I_s(Kr) - \frac{is}{r_0} I_s(Kr) \right\} = \sum A e^{is\theta}$$

By matching terms of $e^{is\theta}$, the coefficients are determined

$$a_s = \frac{-i\, A\, r_0\, f/g}{\frac{\sigma}{f} K r_0 I_s' - s I_s}$$

To evaluate the coefficients, we may use the recursion relationship,

$$I_s'(z) = I_{s+1}(z) + \frac{s}{z} I_s(z)$$

At the very small value of $\sigma/f = 1/710$, the coefficients are

$$a_0 = -i\, A\, r_0\, f/g \Big/ \frac{\sigma}{f} KR_0\, I_1 \, (Kr_0)$$

$$a_1 = i\, A\, r_0\, f/g \Big/ s\, I_s \, (Kr_0)$$

A reasonable and convenient case is a basin of depth 100m at latitude 55°, so $K = f/\sqrt{gh} = 1/265$ km. We may choose the basin radius $r_0 = 265$ km, making $Kr_0 = 1$. Using the tables of Abramowitz and Stegun (1965), the amplitudes of the modes may be computed explicitly. The s = 0 term dominates with $a_o I_o(1) = 2592\ e^{-i\,\pi/2} A$ being 1800 times greater than the s = 1 term $a_1 I_1(0)$. The terms $a_s I_s$ go as $1/s$.

In the s = 0 mode, the entire basin oscillates up and down in unison. Both sea level and current are independent of azimuth. The sea level variation is

$$\zeta = a_o\ I_o\ (Kr)\ e^{-i\sigma t}$$

and the azimuthal current is

$$v = \frac{g}{f}\ \zeta_r = \frac{g}{f}\ a_o\ K\ I_1(Kr)\ e^{-i\sigma t}$$

Tha maximum sea level is found on the boundary. The amplitude at the center of the basin is a_o, since $I_o(0) = 1$. The maximum current is also found on the boundary, but the current diminishes to zero at the center, since $I_1(0) = 0$. Although the details of this model are only a very rough analog to any real situation, the results show explicitly that the factor σ/f greatly enhances the lowest mode of oscillation and minimizes the higher order terms, thus tending to make sea level constant along the coast line.

If a small pole tide current flows from the North Sea, through the Kattegat and into the Baltic, this model applies. In both the North Sea and the Kattegat, quite a striking enhancement over the equilibrium amplitude is observed. It is not implausible that pole tide currents are associated with this enhancement. For example, consider a current u_o = .01 cm/sec flowing through an inlet of width $r_o/10$ (26 km). Then $A = 10^{-3}$, and the amplitude of the sea level variation on the boundary is 2.6 cm. The observed amplitudes in the Baltic area are of order 3 cm. At the boundary the calculated radial current is .36 cm/sec.

The hypothesis of a small current flowing into a basin is consistent with the observations in the Baltic, and pole tide currents flowing from west to east through the shallow seas of Northern Europe are consistent with the phase information of the observations. However, a pole tide current alone cannot be the whole story behind the observed pole tide amplitudes. From the most basic mass conservation standpoint, ignoring any small scale dynamics at this low frequency, mass flow into a basin will cause a general rise in sea level. In a simplified fashion, we may consider the seas of Northern Europe to be a two-basin system, with a wide-mouthed North Sea connected to the Baltic Sea by a narrow inlet. In the inviscid case, equilibrium is obtained when the sea level is the same in both basins. Since the basins are of comparable area ($.575 \times 10^6$ km^2 for the North Sea, $.422 \times 10^6$ km^2 for the Baltic Sea, according to Defant (1961)), the small current entering through the wide mouth of the North Sea would have to be associated with a much greater current (of order 100 times greater) through the narrow inlet to the Baltic Sea. If frictional effects are important, they will act most strongly in the region of high current, causing it to be reduced. Hence sea level variations in the Baltic would be no greater than in the North Sea, which is in contradiction to the observations.

E. Alternative Mechanisms

1. A Helmholtz Resonator Analysis

The uniformity of the observations in the Baltic in space, and the simultaneity in time suggest a Helmholtz resonator approach. When the physical dimensions of a system are small compored to a wavelength, we may make a lumped circuit electrical analogue to its behavior. Following Morse (1948), we associate pressure ($P = \rho g \zeta$) with voltage V, flow (ϕ = velocity x area) with current I, and the ratio of pressure to flow with impedance Z. An ocean basin stores potential energy, acting as a capacitor. Kinetic energy is stored in a channel, as a pressure difference accelerates fluid through it. Along with this inductive behavior in a channel one may expect to find frictional dissipation proportional to the flow.

We may consider a basin with a narrow inlet. The analogue is a circuit with a voltage source, V_o, a resistor, R, and inductor, L, in series, and a capacitor, C, to complete the loop. We want to study the gain and phase of the sea level in the basin, as compared to the sea level outside the inlet, i.e., the voltage across the capacitor as compared to the source. Note that when the circuit is "tuned", so that the impedance of the inductor (inlet) matches that of the capacitor (basin), the only impedance which is felt is due to the resistor, and resonance occurs. The voltage at the capacitor is

$$V_c = \frac{V_o}{1 - (\omega/\omega_o)^2 - iRC\omega}$$

where the driving is at frequency ω, and the resonant frequency is $\omega_o = 1/ LC\ g$ which turns out to be

$$\omega_o = 1/\sqrt{\frac{Al}{ag}}$$

The basin has area A, the channel has length l and cross-sectional area a. Low resonant frequencies will be due to large basins with long and narrow inlets leading to them.

Let us estimate the resonant frequency of the Baltic system. The surface area $A = .575 \times 10^6\ km^2$ (Defant, 1961). We may approximate the inlets of the Oresund and the Store Baelt as of order 20 km x 10 m in cross-sectional area, and 100 km long. The resonant frequency is then $\omega_o = .59 \times 10^{-5}\ sec^{-1}$ (12.3 days period), as compared with the much lower pole tide frequency $\omega = 1.67 \times 10^{-7}\ sec^{-1}$. Even if we extend our idea of a narrow inlet to include the Kattegat and the Skaggerak, of total length 500 km and cross-sectional area of order 80 km x 20 m, we add only a small inductor in series, lowering the resonant frequency to $.46 \times 10^{-5}\ sec^{-1}$ (15.7 days). The impedance of the capacitor dominates that of the inductor at low frequency, and the characteristic leak time of the resistor-capacitor circuit is RC of order 80 days, when the coefficient of friction R is chosen to be of the order of the pole tide frequency. The frequency of the pole tide is about 30 times smaller than the resonant frequency of the Baltic system. This is the regime in which the response of the basin is identical to the input. Thus a Helmholtz resonance does not explain the large amplitude of our observations in the Baltic.

2. Kinematic Response to a Moving Sea Bottom

Let us consider a kinematic, rather than dynamic process, for enhancing the height of the pole tide. Since the Chandler wobble involves the motion of the solid earth, it is natural to examine the response of a small basin of water to a moving sea bottom. Since even the ordinary solid earth tides, M_2 etc., follow an equilibrium law quite closely, we are safe in claiming that the motion of the solid earth must surely be an equilibrium response to the applied Chandler wobble potential U. In that case, the earth moves a distance hU/g in the direction normal to the geoid, where h = .59 is a Love number characteristic of the deformation of the earth.

The equilibrium motion of a concave sea bottom will produce a slight squeezing effect on its contents. What originally was the shore line will later be underwater. For example, consider a conical basin, with sides that slope upwards at an angle α from the horizontal. The bottom moves a distance δ in the normal direction. A geometrical construction shows that the water above the old shore line has the height

$$\zeta = \frac{\delta}{\cos\alpha} - \delta\cos\alpha$$

For a gradual slope, $\cos\alpha = 1 - \frac{1}{2}\alpha^2$ and

$$\zeta \doteq \delta\,\alpha^2 .$$

The sea level change is quite negligible, for the gradual slopes (10^{-3} to 10^{-4}) that are typical over basin bottoms.

3. An Internal Wave Conjecture

Indulging in a backwards approach to science, we may ask what sort of physics may lead to a large amplitude pole tide. We may think of a travelling wave resonance, in which the equilibrium pole tide moves from west to east at a speed which is close to a natural speed of the medium. At about 60 degrees latitude, the pole tide moves around the globe at a rate of .45 m/sec. From rough estimates taken from the Atlas of the North Sea and the Baltic, by the Deutsche Seewarte (1927), first mode internal wave speeds off the Dutch coast vary from .3 to .54 m/sec from February to August.

It may be shown that the particular solution to a forced internal Kelvin wave problem (current normal to the boundary identically zero) is proportional to the equilibrium, with a constant of proportionality α depending on the phase speed c of the medium, and the phase speed c_e of the equilibrium tide, where

$$\alpha = \frac{c_e^2}{c^2 - c_e^2}$$

The observed range of speeds on the Dutch coast would give an $|\alpha|$ of no less than 2.

However, the analysis is not complete without accounting for the homogenous solution and the boundary conditions for a specific problem, which may act to keep the tide equilibrium. More fundamentally, it is difficult to understand how an organized internal water motion may persist over a period of 14 months. The importance of internal motion remains only a conjecture, especially in view of the fact that a large motion at the thermocline corresponds to only a small one at the surface, by a ratio of order 100 to 1.

F. Geophysical Consequences

1. The Damping of the Chandler Wobble in the Ocean

Munk and MacDonald (1960), pages 167-8, estimate the rate of dissipation of energy in the Chandler wobble to be $10^{15}/Q$ erg/sec. For Q's of order 100 to 30, this is a dissipation of 10 to 33×10^{12} erg/sec. An estimate of the contribution of the pole tide to the dissipation of the Chandler wobble has been made by Jeffreys (1970), pages 323-5, and has been discussed by Munk and MacDonald (1960), pages 171-2. By comparing the pole tide to the semi-diurnal tide, and rejecting the possibility of magnification in shallow water, the result is a dissipation of order 10^{10} erg/sec., about 3 orders of magnitude below the "observed" dissipation rate.

Without a successful model of the pole tide in shallow basins and shelves, it is difficult to made a definitive statement on the damping of the wobble by the ocean. However, the large amplitudes in Northern Europe, and the continuum levels which are higher than equilibrium throughout the world suggest that some new order of magnitude estimates are in order. For example, let's follow up the consequences of the circular basin with flow through an inlet problem, where we had an average current of order .2 cm/sec flowing around the basin. We may consider the linearized bottom stress τ, as discussed in the section "The problem of variation along coast lines", above. The rate of dissipation for the whole basin is the time average $\langle \tau v \rangle$ over a pole tide period, times the area of the basin,

$$\frac{\partial E}{\partial t} = .0025 \, |\upsilon| \, \frac{\upsilon^2}{2} \text{ area}$$

The tides in the Baltic are quite small, of order 10 cm, as opposed to order 100 cm in the North Sea. M_2 tidal currents of order 50 cm/sec are reported from the light vessels off the Dutch coast (Otto, 1964), so we may estimate tidal currents of order υ = 1/10 x 50 cm/sec = 5 cm/sec in the Baltic. With an area of .422 x 10^6 km^2, the Baltic rate of dissipation under this model would be 10^{12} erg/sec, two orders of magnitude greater than the previous estimate for the whole world. One might suppose that the North Sea would have similar dissipation. The total area of continental shelves and shallow seas is 27.49 x 10^6 km^2 (Defant, 1961). For tidal currents from 10 to 50 cm/sec, and for pole tide currents of .01 to .1 cm/sec, the range of dissipation is .3 to 170 x 10^{12}. Thus it is within the realm of possibility that the pole tide in shallow water can account for all the dissipation of the Chandler wobble, although this result cannot be substantiated at the present time.

2. The (non-) Excitation of the Chandler Wobble by a Small Sea

A pole tide is observed in the shallow seas of Northern Europe, with an amplitude an order of magnitude greater than equilibrium. What influence may this local oscillation have on the wobble as a whole?

Following the lead of Munk and MacDonald (1960), p. 155, the wobble excitation functions due to the cross products of inertia from the vertical oscillation of the water mass are

$$\phi_1 = \bar{\Phi}_1 + \phi_1' = \frac{c_{13}}{C-A} = \frac{-R_e^4 \rho_w}{C-A} \iint \{(\bar{\zeta} + \zeta') \sin\theta \cos\theta \cos\lambda\} \sin\theta \, d\theta \, d\lambda$$

$$\phi_2 = \bar{\Phi}_2 + \phi_2' = \frac{c_{23}}{C-A} = \quad \text{"} \qquad \text{"} \qquad \sin\lambda \qquad \text{"}$$

where $\bar{\zeta}$ is the equilibrium tide, and ζ' is the deviation from equilibrium. The major principal moment of inertia is $C = .333\ M_e R_e^2$, and A is a minor principal moment. The wobble excitation contribution from the equilibrium tide may be expressed in terms of the rotation pole itself. After Haubrich and Munk (1959)

$$\sigma_r \bar{\Phi}_1 = T_1 \Omega m_1 + R \Omega m_2$$

$$\sigma_r \bar{\Phi}_2 = T_2 \Omega m_2 + R \Omega m_1$$

where σ_r is the wobble frequency of a rigid earth, $(C-A)\Omega/A$. The terms T_1, T_2 and R are evaluated by integrating the equilibrium tide over an earth with continents specified by the "Ocean function", as descrived in Appendix 1 of Munk and MacDonald (1960). The resulting equations of motion describe a coupled linear oscillator, with forcing due to the non-equilibrium tide

$$m_{1t} + \sigma_e m_2 - \sigma_r \bar{\Phi}_2 = \sigma_r \phi_2'$$

$$m_{2t} - \sigma_e m_1 + \sigma_r \bar{\Phi}_1 = -\sigma_r \phi_1'$$

where σ_e is the wobble frequency of an elastic earth without oceans. If we define a local non-equilibrium tide over an area $\Delta\theta\Delta\lambda$ at colatitude θ_o, longitude λ_o, with amplitude ζ'_o,

$$\zeta' = \zeta'_o \Delta\theta\Delta\lambda \; \delta(\theta-\theta_o, \lambda-\lambda_o)$$

then the forcing functions are

$$\sigma_r \Phi'_1 = -\frac{\Omega R_e^4}{A} \rho_w \;\; \zeta'_o \;\; \Delta\theta\Delta\lambda \sin^2\theta_o \cos\theta_o \cos\lambda_o$$

$$\sigma_r \Phi'_2 = \qquad \text{"} \qquad\qquad \text{"} \qquad\qquad \sin\lambda_o$$

When we consider the shallow seas of Northern Europe, not far from the Greenwich meridian, we may set $\lambda_o = 0$ for a first approximation, and thus consider only the Φ'_1 excitation term. In this case, the second order equation of motion becomes

$$m_{2_{tt}} + \sigma_o^2 \; m_2 = \sigma_r \Phi'_{1_t} - R\Omega \; \sigma_r \Phi'_1$$

where the free oscillation of an elastic earth with an equilibrium pole tide has the frequency σ_o,

$$\sigma_o^2 = (\sigma_e - T_1\Omega)(\sigma_e - T_2\Omega) - (R\Omega)^2$$

A harmonically forced oscillation, at the frequency σ, gives the solution

$$m_2 = \sigma_r \Phi'_1 \; \frac{\left(\sigma^2 + (R\Omega)^2\right)^{\frac{1}{2}}}{\sigma^2 - \sigma_o^2} \; e^{i \tan^{-1}(\sigma/R\Omega)}$$

Let us now make an estimate of the excitation function $\sigma_r \Phi'_1$. For our purposes $A \cong C = .333 \, M_e R_e^2$, so that the constant term $\Omega \rho_w R_e^4 / A \cong \Omega \rho_w R_e^2 / .333 \, M_e = 1.49 \times 10^{-14}$ (cm sec)$^{-1}$. At longitude zero and colatitude 30 degrees, the geographical term is $\sin^2\theta_o \cos\theta_o = .217$. The combined area of the Baltic and North Seas is $.422 + .575 \times 10^6$ km^2, as compared to the total surface are of the globe $4\pi R_e^2 = 512.9 \times 10^6$ km^2. That is, we are interested in only .19 per cent of the globe, so $\Delta\theta\Delta\lambda$ = $.0019 \cdot 4\pi = 2.45 \times 10^{-2}$. An amplitude $\zeta_o^1 = 3$ cm gives us

$$\sigma_r \Phi_1^1 = -2.38 \times 10^{-16} \text{ sec}^{-1}$$

Taking the Haubrich and Munk (1959) estimate of $R = -.075 \times 10^{-4}$, we see that $\sigma/R\Omega = -1/300$, where the Chandler frequency is $\sigma = 1.67 \times 10^{-7}$ sec^{-1}. Thus

$$m_2 = -2.38 \times 10^{-16} \frac{\sigma}{\sigma^2 - \sigma_0^2}$$

The observed polar motion is of order $m_2 = 3$ meters$/R_e = .52 \times 10^{-6}$. To produce that amount of motion, the frequency of the local oscillation would have to be extremely close to the Chandler frequency, by one part in 10^6. Thus the coupling between the locally enhanced pole tide and the wobble motion of the earth is far too weak to be significant.

G. Discussion of the Behavior of the Continuum

There are two possible mechanisms for the low frequency sea level continuum power in northern Europe. There may be local, direct atmospheric forcing, or there may be a wave-like radiation due to a disturbance generated some distance away. In the first case, the phase velocity of the sea level variations will be that of the forcing functions. In the second case, the phase velocity will be the characteristic speed of the medium. Since we have ruled out small scale internal and barotropic waves, we rule out slow speeds of propagation, retaining only the basic gravity wave with speed $\sqrt{gh}$. The speeds deduced from coherency phase information in the Baltic are consistent with a gravity wave propagation across the Baltic. But the speeds deduced from stations in the North Sea and the Kattegat are much too slow for a gravity wave, and hence one must think of direct forcing by the atmophere.

In considering direct forcing, we fall back to the ubiquitous $F = ma$, in the sense that the smaller the mass of water in an ocean basin, the more dramatic will be its response to a particular applied force. Thus we would expect the response of a basin to be inversely proportional to its depth. The power of the continuum in the Baltic is more than twice as large as in the North Sea, and the depth is about half as great. According to Defant (1961), the mean depth of the Baltic is 55 meters, while that of the North Sea is about 94 meters. Furthermore, the continuum power at Cascais is only about one tenth as great as that of the Baltic. Cascais is situated on the narrow continental shelf in Portugal, with depths of order 200 to 1000 meters.

In following this idea further, the observed weakness of the low frequency atmospheric pressure variations, and the observed contrast between the signatures of pressure and sea level across Northern Europe, tend to make one skeptical of the importance of direct atmospheric forcing on the North Sea and the Baltic. However, we have neglected an important topic. The work of Wunsch (1972), with an eight year record of sea level, atmospheric pressure and wind at Bermuda, suggests that at low frequency (about 1-2 months period), the influence of wind on sea level surpasses that of atmospheric pressure. The present analysis is restricted to periods longer than 2 months, and hence more attention to wind effects is in order. Unfortunately, no tables of "mean monthly winds" for the years 1900-64 exist, which could be compared directly with the sea level records already analyzed. One might attempt to estimate mean geostrophic winds from a least-squares fit to pressure gradients between stations in an array across Northern Europe. It must be emphasized that extreme caution must be exercized in the interpretation of the algabraic differences between two separate time series, subject to random error and local noise. At the very least, such a task would require a preliminary test study on long simultaneous original records of pressure, sea level and wind, and is beyond the scope of the present project. At this stage of the analysis, where we have neglected the wind effect, we can not prove that the atmosphere directly forces the observed highly coherent sea level behavior, but it appears to be the best guess.

H. Assessment of the Pole Tide Interpretation Problem

The cause of the enhancement of the pole tide in the shallow seas of northern Europe must be considered to be still unknown. A small-scale resonance is ruled out by the constraints of the low frequency boundary conditions, the sharpness of the observed spectral peak despite the existence of mean currents and transient events, and by the lack of small scales in the observations. An internal Kelvin wave is ruled out by the same reasons, as well as by the large internal amplitude necessary to sustain even a small amplitude at the free surface. The effect of the equilibrium motion of the sea bottom is negligible. A very small pole tide current flowing into a basin would cause an enhancement of amplitude. But since the frequency is too low for a Helmholtz resonance, it is not clear how a simple current flow mechanism would allow sea level to be higher in the Baltic than in the North Sea, as observed. Furthermore, the possible source of a pole tide current, by topographic interaction with the equilibrium tide or some other mechanism, is an unsolved problem. Since slowly moving internal and small-scale barotropic modes are ruled out, it is difficult to see how a pole tide current could satisfy the slow phase speed of the observations. The situation is "appallingly uncertain."

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